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PROCEDURES FOR PREDICTIVE ANALYSIS OF SELECTED HYDROLOGIC IMPACTS OF SURFACE MINING

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Procedures for determining the transmissivity of coal and overburden aquifers from single-hole aquifer tests are provided. The procedures permit the analysis of recovery data, affected by well-bore storage, following a prolonged pumping period. Well-bore storage is an important effect in the recovery of low transmissivity aquifers often encountered in coal mining related hydrology. Several approximate, closed form formulas for estimating selected impacts of surface mining on ground water are provided. Among them are formulas for estimating ground-water inflows to an advancing pit and to a pit advancing parallel to an alluvial valley. Formulas for calculating the extent of the depressed piezometric surface as a function of time and distance from the pit are developed. These formulas can be used to assess the probable severity of corresponding impacts and to judge the need for additional data and more detailed models in site specific situations.

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FOREWORD

When energy and material resources are extracted, processed, converted, and used, the related pollutional impacts on our environment and even on our health often require that new and increasingly more efficient pollutional control methods be used. The Industrial Environmental Research Laboratory - Cincinnati (IERL - Ci) assists in developing and demonstrating new and improved methodologies that will meet these needs both efficiently and economically.

This report presents a methodology for predicting the post-mining salt load in combined direct and subsurface runoff from a watershed partially disturbed by surface mining. Also provided are methods for analyzing single-well test data collected in tests of the low transmissivity aquifers often encountered in coal mining related hydrology. Methods are developed for predicting several hydrologic impacts of surface mining.

David G. Stephan
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ABSTRACT

The purpose of this report is to present a methodology for the prediction of selected hydrologic impacts of surface coal mining. Procedures for estimation of the chemical and hydrologic parameters required by an algebraic water quality model are provided. The model predicts the long term mean dissolved solids concentration in combined direct and subsurface runoff from a watershed partially disturbed by mining. Use of the model requires that precipitation be partitioned into direct runoff and infiltration. This is accomplished in this report using the historical record of daily precipitation and the estimation of direct runoff by the Soil Conservation Service Curve Number method. Long term mean values of monthly infiltration are utilized in a soil-water balance to estimate mean annual subsurface runoff. The resulting estimates of mean annual direct and subsurface runoff from both the mined land and the undisturbed portion of the watershed permit computation of the hydrologic parameters in the algebraic model. The computational procedure is demonstrated in a step-by-step calculation for a mine site in Colorado. The predicted results are in satisfactory agreement with short term (2 and 3 year) observations.

Procedures for determining the transmissivity of coal and overburden aquifers from single-hole aquifer tests are provided. The procedures permit the analysis of recovery data, affected by well-bore storage, following a prolonged pumping period. Well-bore storage is an important effect in the recovery of low transmissivity aquifers often encountered in coal mining related hydrology. Several approximate, closed form formulas for estimating selected impacts of surface mining on ground water are provided. Among them are formulas for estimating ground-water inflows to an advancing pit and to a pit advancing parallel to an alluvial valley. Formulas for calculating the extent of the depressed piezometric surface as a function of time and distance from the pit are developed. These formulas can be used to assess the probable severity of corresponding impacts and to judge the need for additional data and more detailed models in site specific situations.

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SECTION 1

INTRODUCTION

Federal and State regulations require an analysis of the potential influence of coal mining upon the hydrologic balance in the area affected by mining. Potential effects of coal mining upon the hydrologic balance include changes in the quality of ground and surface waters and a modification of the relative quantities of direct and ground-water runoff. Other possible effects are the modification of recharge to regional and local aquifers, a change in the pattern of ground-water flow, and a shift in the magnitude and peak of the runoff hydrographs. The changes that may be anticipated are different in the active mining phase than in the long term, post-mining phase.

Implicit is the requirement that the influence of a particular mining project upon the hydrologic balance be predicted before mining is initiated. This can be accomplished only through the use of models, even if they are conceptual. Each of the several components of the hydrologic balance is a complex phenomenon that exhibits all of the vagaries of natural processes, and models range from simple, non-quantitative concepts through sophisticated stochastic models to detailed, physically based descriptions. Those that are faced with the preparation and review of predictions relative to the hydrologic consequences of mining must select methods or models upon which to draw conclusions. The most useful set of models is that which provides results in the desired form and of suitable reliability and is, at the same time, commensurate with the experience, technical knowledge, resources, and data that can reasonably be obtained by the user.

In keeping with this perception, this report presents a set of methods by which the influence of surface coal mining upon the hydrologic balance can be analyzed. The methods presented in this report are not applicable in all situations, of course, nor are they intended to be. Through the use of examples, the application of the methods are demonstrated. It is anticipated that interested readers will devise ways to modify the procedures for site specific needs. It is hoped that a reasonable balance has been struck between the degree of rigor and realism in the methods and the knowledge, resources, and data required to apply them. The emphasis throughout the report is on guidelines for application rather than on theoretical justification.

Section 4 develops a method by which the influence of surface mining on the partitioning of mean annual watershed drainage into direct runoff and ground-water runoff can be estimated in the post-mining phase. It is shown how these results can be used to predict the mean annual dissolved solids content in total watershed drainage. A detailed example computation is included that utilized data from a site in Colorado. Also available for this site are observations that permit a comparison of predicted results with measured values. The agreement is found to be satisfactory for this particular area.

An important aspect of mine hydrology during the mining phase is the prediction of mine inflows from disturbed aquifers, the disturbance of the piezometric surface, and effects on nearby aquifers such as those in alluvial valleys. These aspects are of concern relative to water quality as well because of the possible need to discharge the inflows in surface waters. Fundamental to the prediction of these phenomena is knowledge of the hydraulic coefficients of the aquifers. Possibly the greatest single cost of a hydrologic impact analysis is the construction of wells to be used for aquifer testing and monitoring. In many cases the cost can be substantially reduced by the utilization of single-well aquifer tests. A procedure for the analysis of recovery data in pumped wells is developed and demonstrated in Section 5. Finally, procedures for calculating mine inflows for several different situations are developed and demonstrated in Section 6.

SECTION 2

CONCLUSIONS

A major purpose of this report is to present a method by which the chemical and hydrologic parameters required for use in an algebraic water quality model can be obtained. At the outset, the intent was to prepare a set of charts and nomographs that could be used in conjunction with basic watershed characteristics to determine the necessary parameters. This proved to be less feasible than the method presented herein.

The algebraic water quality model referred to above and elsewhere in this report requires the estimation of the mean concentration of dissolved solids in combined direct and subsurface runoff from the undisturbed area (P_n), the mean TDS concentration in direct runoff from the mined land (P_{sm}), and the mean concentration in subsurface runoff from the mined land (P_{gm}). The value of P_n is most reliably determined from premine monitoring on the watershed to be mined. On the assumption that the mined land will be reclaimed, the best estimate for P_{sm} is the mean concentration of dissolved solids in direct runoff from the undisturbed area. Again this can be determined from pre-mine monitoring. The parameter P_{gm} is more difficult to estimate. Dissolved solids data for spoil waters in nearby areas with a similar hydrogeochemical environment will probably provide the most reliable estimate for P_{gm}. The value of P_{gm} can be estimated from the dissolved solids concentration in saturated extracts prepared from drill cuttings or core samples of the overburden. Large variations in individual determinations are to be expected and many determinations may be required to produce a meaningful average value.

The model also requires estimation of two hydrologic parameters, K and f_{sm}, both of which require estimation of the mean partitioning of precipitation into direct runoff and infiltration. It is concluded that the Soil Conservation Service Curve Number method can be used for this purpose. This method strikes a reasonable balance between the tradeoff of improved simulation of the physical processes involved and the cost in terms of data

requirements. Calculation of daily rainfall excess from the historical precipitation record is required. The curve number method can be applied for each day of the historical record or a histogram method can be used to reduce the number of computations. The estimates of long term mean monthly rainfall excess by the histogram method agreed closely with those calculated on a daily basis.

The difference between precipitation and precipitation excess is used as input to the soil zone. A water balance for the root zone is used to estimate the quantity of subsurface runoff. It is concluded that this method yields satisfactory estimates of long term mean subsurface runoff if the time increment for the balance is no longer than one month. The computation is sensitive to the value of available water holding capacity of the root zone.

The algebraic water quality model was used to estimate the mean water quality in combined direct and subsurface runoff for an area in Colorado where data were available for comparison with the predictions. The predicted value of P_t was within the range of values observed for either two or three years on 5 subwatersheds in the mining areas. Prediction of combined direct and subsurface runoff agreed satisfactorily with the observed value, also.

Sections 5 and 6 of this report pertain to the development of procedures that may be used to estimate hydraulic properties of coal or overburden aquifers and to predict hydrologic impacts of mining for selected problems. It is concluded that single hole recovery data taken following a prolonged pumping period can be analyzed conveniently by either of two methods developed in this study. Both methods account explicitly for the effects of well bore storage that are common in tests of aquifers with low transmissivity.

Approximate closed-form formulas for several ground water problems commonly encountered in coal mining hydrology are developed. These formulas can be used for predicting pit inflows for a few different situations, the extent to which the piezometric surface will be disturbed, and some aspects of spoil bank hydraulics. The analytic models are simple and have minimum data requirements relative to the many numerical models that are available. However, their use is constrained by the degree of idealization of the flow problems, particularly with respect to the requirement that the aquifers be homogeneous. The formulas can be used to assess the probable severity of corresponding hydrologic impacts and to judge the need for additional data and more detailed models in site specific instances.

SECTION 3

RECOMMENDATIONS

The following specific recommendations relate to the use of the algebraic water quality model discussed in Section 4.

1. The algebraic water quality model is recommended for use when the long term mean concentration of dissolved solids in combined direct and subsurface runoff is required. The model is not appropriate for predicting temporal fluctuations (e.g. seasonal changes).

2. Determination of the parameter P_n should be accomplished by pre-mine monitoring. P_n is the discharge weighted, mean concentration of dissolved solids in combined direct and subsurface runoff from the watershed to be mined. It is recommended that the station at which P_n is to be determined be located on a watershed drainage channel where there is reasonable assurance that all direct and shallow subsurface runoff will be observed. Subsurface runoff via deep regional aquifers is not considered in the model. Continuous records of stream flow as indicated by continuous recordings of stage on a rated section are recommended. Samples for concentration measurements should be collected with a frequency sufficient to define the seasonal fluctuation of dissolved solids concentration. This may require daily samples during periods of peak seasonal runoff. Streams that flow only in response to individual precipitation events should be sampled so that the concentration variation over the runoff period is defined.

3. The parameters P_{sm} and P_{gm} are the mean concentrations of dissolved solids in direct runoff and subsurface runoff, respectively, from the mined land. It is recommended that P_{sm} be taken as being equal to the concentration of direct runoff from the undisturbed land. This recommendation is based on the assumption that the reclaimed lands will exhibit soil chemical and erosive characteristics similar to those existing prior to mining. The parameter P_{gm} can be estimated from the dissolved solids concentration in saturation extracts prepared from overburden drill cuttings and cores. A great deal of spatial variation (both vertically and areally) is to

be expected and it is recommended that sufficient determinations be made to provide a statistically reliable mean. The user of this technique should realize that the method may provide only a rough estimate. Any existing data on spoil-water quality from similar hydrogeochemical settings should be examined and used as a guide to the determination of P_{gm}. Where such data exists, its use is preferable to the saturation extract method.

4. The algebraic model requires that the unit area ratio of total runoff on the undisturbed portion to that on the mined land be estimated. This is the parameter K in the model. Also required is f_{sm}, defined as the fraction of total runoff from the mined land that is direct (surface) runoff. Estimation of both K and f_{sm} require that the mean annual partitioning of precipitation into direct runoff and infiltration be estimated. This report recommends the use of the Soil Conservation Service Curve Number methods for this purpose. This method is recommended, not because of any inherent superiority, but because it permits calculation of rainfall excess from data that are usually available. It is recommended that rainfall excess be calculated on a daily basis from at least 20 years of daily precipitation data.

5. A root zone water balance calculation is recommended for estimation of the quantity of subsurface runoff. It is recommended that the time increment for the water balance calculation be no greater than one month.

Sections 5 and 6 of this report deal with the use of analytic procedures for determination of aquifer hydraulic properties and calculation of aquifer response to mining. The following recommendations relate specifically to these results.

6. It is recommended that single hole aquifer tests be conducted by pumping up to several hours and that emphasis be placed on the collection and analysis of recovery data. Prolonged pumping creates a cone of influence that is substantially larger than that created by a slug test and the recovery data reflect the response in a larger volume of aquifer. It is recommended that the Papadopulos-Cooper theory, as extended in this report to the recovery period, be used to estimate the hydraulic properties from recovery data. Alternatively, an algebraic method not requiring a curve match may be used. Both methods are useful only when well-bore storage is important.

7. Several problems of aquifer response to mining were analyzed by approximate analytic techniques. The resulting formulas are recommended for use when there is reasonable assurance

that the idealized situations used in the analyses approximate the field problem. Probably most critical in this regard is the idealization of the aquifer as homogeneous with respect to hydraulic conductivity.

SECTION 4

THE QUALITY AND QUANTITY OF RUNOFF FROM SURFACE MINED WATERSHEDS

Estimation of the effects of surface mining on the hydrologic balance of a watershed requires the prediction of both the quality and quantity of runoff from the disturbed watershed. The quantity of runoff from a watershed depends upon such factors as:

1) interception and evaporation of precipitation from the surfaces of vegetal cover,
2) depression storage in surface puddles, ditches, and other surface depressions,
3) infiltration of precipitation into the soil,
4) percolation of water through the soil under the influences of gravity and capillarity,
5) near surface perching of subsurface waters and movement of these waters to drainage channels,
6) deep percolation and ground-water flow,
7) overland flow,
8) evapotranspiration,
9) intensity and duration of storms.

A set of even more fundamental factors affecting the runoff can be listed under each of the above. For example, infiltration is influenced by permeability and antecedent moisture conditions. Evapotranspiration is affected by temperature, solar radiation, stage of plant growth and several other variables. Likewise, a list of factors affecting the quality of runoff can be prepared and would include such variables as:

1) the geo-chemical characteristics of solids contacted by water and the distribution of these solids,
2) resident times and rates of chemical reactions,
3) weathering agents such as freeze-thaw cycles and microbial activity.

Each of these parameters depends upon a host of more fundamental variables.

Models of the post-mining water quality hydrology could range from very detailed models that attempt to account for the influence of all fundamental variables upon each of the components, to simple formulations in which the effects of many variables are neglected and the influence of others is included only implicitly in the form of lumped parameters. Usually, the

generality, confidence, and reliability of the more detailed models are obtained at the expense of increased complexity and increased data requirements relative to the simpler models. Within the context of routine use by practioners, the "best" model is that which will yield results of acceptable accuracy and reliability at a minimum cost. Obviously, the model selected for use will depend upon the requirements of the application, the available data, the available resources, time contraints, and the knowledge and experience of the user.

Consistent with the approach used elsewhere in this report, the developments of this section were obtained by attempting to strike a reasonable balance between the perceived requirements and constraints of the prospective user and the generality of the predictive procedures that are presented.

A COMBINED WATER AND SALT BALANCE

Of principal interest is the change in the water quality hydrology that results from disturbing a portion of a watershed by surface mining. The change will occur as a result of a modification of one or more of the previously listed factors that affect the water quality hydrology. Based upon a simple water and salt balance approach, the dependence of the mean concentration of dissolved solids in total watershed runoff upon several of the more important factors has been derived previously (McWhorter and Rowe, 1976; Rowe and McWhorter, 1978; and McWhorter, et al., 1979). The equation is

$$P_t = \frac{KRP_n + P_m}{1 + KR} \qquad (1)$$

where P_t = mean concentration of dissolved solids in total watershed runoff including both direct and subsurface runoff,

P_n = mean concentration of dissolved solids in combined direct and subsurface runoff from natural (undisturbed) portion of the watershed,

P_m = mean concentration of dissolved solids in combined direct and subsurface runoff from mined land,

K = the ratio of total runoff per unit area (including direct and subsurface runoff) on the undisturbed area to that on the mined land,

R = the ratio of the area of the natural land to the area of the mined land.

The material balances also lead to

$$P_m = f_{sm}P_{sm} + (1-f_{sm})P_{gm} \quad (2)$$

and

$$P_n = f_{sn}P_{sn} + (1-f_{sn})P_{gn} \quad (3)$$

where f_{sm} = the fraction of runoff from mined land that is direct runoff,

f_{sn} = the fraction of runoff from natural (undisturbed) land that is direct runoff,

P_{sm} = concentration of dissolved solids in direct runoff from mined land,

P_{gm} = concentration of dissolved solids in subsurface runoff from mined land,

P_{sn} = concentration of dissolved solids in direct runoff from natural land,

P_{gn} = concentration of dissolved solids in subsurface runoff from natural land.

The above equations are dimensionally consistent and any consistent set of units can be used. In this report concentrations are expressed in milligrams per liter (mg/ℓ). Volumes of runoff are expressed in cubic meters or, when referred to a unit area, in centimeters.

Throughout the report the word runoff will refer to the total runoff. Runoff is comprised of two components; direct runoff and subsurface runoff. Direct runoff is the overland flow that makes its way into a drainage channel and contacts the soil at the surface or within a few millimeters of the surface. Subsurface runoff refers to waters that percolate beneath the root zone of vegetation and eventually reappears in the drainage channels. As used in this report, subsurface runoff may lag direct runoff by times much greater than those usually invisioned in storm hydrograph analysis. The term interflow is not used for this reason. Interchange of waters in the near-surface system with deep, regional ground waters is not considered.

The above equations were derived upon the assumption that the net change in the volume of water stored in the watershed is zero over the time period of interest. Relative effects of depression storage, infiltration, and evapotranspiration upon total runoff for each portion of the watershed are lumped into the parameter K. In arid and semi-arid areas, evapotranspiration is limited by the availability of water in the root zone. Therefore, factors which affect the partitioning of precipitation into direct runoff and infiltration in turn affect

evapotranspiration. The partitioning of total drainage into direct and subsurface runoff is accounted for in both K and the f factors. Also, it is assumed that precipitation is uniform over the watershed. It is emphasized that the above equations do not apply to particular precipitation events, but rather, to long-term average conditions.

Previous use of the above equations has been to estimate the contribution of disturbed land to the total observed salt load from a watershed. This was accomplished by applying the equations to individual watersheds comprising a partially mined area and using regression techniques to determine the values of K and f. To be useful in the predictive mode, however, the values of all parameters must be estimated prior to mining. The required parameters are the water quality parameters P_{sm}, P_{gm}, P_{sn}, P_{gn}, and the hydrologic parameters K, f_{sn}, and f_{sm}. The parameter R is the area of the natural land relative to the mined area as is determined from the mining plan. The water quality and hydrologic parameters in the simple model described above depend upon more fundamental quantities, of course. It is the purpose of the following subsections to outline and demonstrate a procedure by which the required parameters may be estimated.

DETERMINATION OF WATER QUALITY PARAMETERS

Equation 3 expresses the dependence of the dissolved solids concentration in runoff from the natural (undisturbed) portion of the watershed upon the qualities and relative quantities of direct and subsurface runoff. This equation is provided and recommended for use in cases where P_n cannot be measured directly. It is anticipated that monitoring in the pre-mining phase will establish the value of P_n directly in most cases. In order to establish a value for P_n by monitoring, it will be necessary to measure the dissolved solids concentration and discharge at the drainage point for the watershed under consideration. The appropriate value of P_n is the discharge weighted mean

$$P_n = \frac{\sum_{i=1}^{n} P_i Q_i}{\sum_{i=1}^{n} Q_i} \qquad (4)$$

where P_i is the dissolved solids concentration in discharge Q_i, and n is the number of determinations. Ideally, P_n is obtained

by continuously monitoring the concentration and discharge, but frequent grab samples will suffice. The frequency of sampling and the time period over which sampling must be maintained are site-specific considerations. Continuous water level recording on a rated section is a necessity. On intermittently flowing streams, several samples from individual runoff events will often be required. On streams in which the major variation in discharge is seasonal, the frequency should be greatest during the high runoff period. It has been the author's experience that the dissolved solids concentration in runoff from undisturbed watersheds decreases as the discharge increases. It is recommended that sampling be sufficient to establish the relationship between P and Q.

A similar procedure for the determination of P_m is not possible in the pre-mining phase. Therefore, it is necessary to estimate P_{sm} and P_{gm} and use Eq. 2 to obtain P_m. Determination of f_{sm} is discussed subsequently. The values of P_{sm} and P_{gm} are the mean concentrations of dissolved solids in direct and subsurface runoff, respectively, from the mined portion of the watershed. A field study of post-mining water quality at a mine in northwest Colorado (McWhorter, et al., 1979) suggests that the concentrations of solids in saturation extracts (Richards, 1954) prepared from drill cuttings in the overburden is a direct indicator of P_{gm}. Elsewhere, however, the dissolved solids concentrations in ground water in the cast overburden has been found to be greater than that in saturation extracts by factors of 2 or 3 (Moran, et al., 1979; Groenewold, 1979; Pegenkopf, et al., 1977). Furthermore, measured concentrations of dissolved solids in water contained in spoil material is highly variable at most locations, with the standard deviations being a large fraction of the mean.

The current state of knowledge relative to the chemistry of waters in spoil notwithstanding, projections of the water quality affected by mining are required in mining permit applications. Probably the most reasonable estimate of P_{gm} can be made from a judicious study of the quality of spoil water from nearby mines in a similar geo-chemical environment. Sampling of springs formed on the interface between the spoil and the undisturbed underburden is possible in some cases, but sampling from wells completed in the spoil aquifer will usually be required. In the absence of this possiblilty, present experience suggests that the dissolved solids concentration in extracts from saturated drill cuttings will provide a reasonable lower limit for P_{gm}. If the dissolved solids derive mainly from highly soluble sodium salts, P_{gm} can be expected to be greater than the saturation extract value, perhaps by as much as a factor of three. It is recognized

that the above statements do not constitute a satisfactory guide and that additional research is needed. The accumulation of data on spoil-water quality now accompanying the expansion of surface coal mining should permit more definitive guides in the future.

Virtually all spoil will be topsoiled on future surface mines. In this case, the most reasonable estimate for the dissolved solids concentration in direct runoff P_{sm} from the topsoiled mine is the value that exists on the undisturbed watershed in which the mine will be located. Thus, P_{sm} can be determined from data collected in the pre-mining phase. Cases may be encountered where thin topsoil placed on the spoil is eroded to the extent that the spoil becomes exposed. Direct runoff in contact with the spoil can be expected to contain dissolved solids in concentrations that differ from that contacting only the soil. However, the readily soluble salts in spoil contacted by direct runoff are rapidly leached because the volume of water contacting the spoil is large relative to the volume contacted. Dissolved solids concentrations in direct runoff from even highly saline materials are surprisingly small.

Before proceeding to a discussion of the hydrologic parameters, a few concluding comments on the quality parameters are in order. Sampling of waters contained in cast overburden has shown the dissolved solids concentrations to be highly variable, even within a single mine. The variability can be attributed to different leaching rates caused by variable permeability and water availability, to differing chemical composition of materials contacted, to variable clay contents and to variable degrees of aeration, among other factors. Clearly, the single parameter P_{gm} is a lumped parameter, the value of which is a consequence of complex interactions among several variables. Furthermore, the dissolved solids concentration cannot be expected to be constant in time. It has been demonstrated in many leaching studies that the dissolved solids in the effluent decrease as the volume of effluent increases. It seems reasonable to assume that a similar improvement will occur in the field. The improvement is expected to be slow, however. On the order of 1 pore volume of throughput is required to effect significant improvement. Particularly in arid and semi-arid regions where subsurface runoff is a few centimeters per year at most, noticeable improvement in P_{gm} cannot be expected for several decades.

DETERMINATION OF HYDROLOGIC PARAMETERS

There are three explicit hydrologic parameters in the model; K, f_{sm} and f_{sn}. The parameter f_{sn} need be estimated only if P_n cannot be successfully evaluated through a pre-mining monitoring program. The following paragraphs describe a means for estimating

K and f_{sm}. The same procedure applies for estimation of f_{sn} should it be necessary.

The parameter K represents the ratio of total runoff per unit area on the undisturbed portion of the watershed to that on the mined portion. In situations where evapotranspiration is limited by water availability, any factors affecting the partitioning of precipitation into direct runoff and infiltration also affect the value of K. Use of Eq. 1 for predictive purposes will be practical only if a practical method for determining K and f_{sm} can be devised. A review of the physical interpretation of K and f_{sm} is in order before proceeding to a discussion of their estimation. In the derivation of Eq. 1 (McWhorter, et al., 1979) it is shown that

$$K = \frac{1 - f_{en}}{1 - f_{em}} \tag{5}$$

where f_{en} = fraction of the precipitation per unit area that is consumptively used on the natural area,

f_{em} = fraction of the precipitation per unit area that is consumptively used on the mined area.

Under the assumption that there is no net change in water stored in the watershed, the numerator of Eq. 5 represents the fraction of precipitation that becomes runoff from the undisturbed portion of the watershed. The denominator is the corresponding quantity for the mined area. Both are on a per unit area basis.

The quantity of water consumptively used will depend upon the type and quality of the vegetal cover, the potential evapotranspiration, and the timing and volume of infiltration into the soil. In arid and semi-arid climates, the potential annual evapotranspiration is larger than the mean annual precipitation. Considering the fact that a fraction of precipitation runs off as overland flow instead of entering the root zone, it becomes apparent that the potential evapotranspiration is an even greater multiple of the volume of soil water available for plant use. At first glance it would seem, therefore, that no subsurface runoff would occur under such circumstances. However, the timing and volumes of infiltration may be such that, at particular times, the water holding capacity of the soil is exceeded and percolation through the root zone occurs. This is especially true where a large fraction of the annual precipitation is in the form of snow that accumulates through the winter and melts in a relatively short time in the spring. Subsurface runoff may occur in response to percolation below the root zone during this period,

even though there exists a deficit of available soil moisture on the average over the year.

Because the potential annual evapotranspiration is greater than the annual available supply in the root zone, the actual evapotranspiration over the year is limited by water availability. Thus, a critical factor in the estimation of actual evapotranspiration is the estimation of the quantities of water that enter the root zone. Clearly, this is directly dependent upon the partitioning of precipitation into infiltration and direct runoff components. Also, the fraction of total drainage from the mined land that is overland flow (i.e. f_{sm}) is directly dependent upon the partitioning of precipitation into infiltration and overland runoff components.

The importance of the partitioning of precipitation into infiltration and runoff components dictated that the focus of this portion of the study be on methods by which this partitioning can be estimated as a function of precipitation, infiltration capacity of the soil, antecedent moisture conditions, and depression and interception storage. Substantial effort went into an attempt to prepare nomographs from which, not only the partitioning could be estimated, but also the values of K and f_{sm}. For this approach to be practical the independent variables used to enter the nomographs must be determinable by inspection of the watershed, coupled with pertinent measurements that are relatively easy to make. Several available hydrologic models were investigated with the intent of generating ground-water and overland runoff data for a wide variety of conditions in which selected key variables would be systematically varied. For these data correlations of the ground-water and overland runoff with the pertinent independent variables would be established. The USDA HL-74 hydrologic model was selected and this approach pursued. A difficulty with this procedure is that the USDA model (and others investigated)are basically oriented toward hydrograph prediction, while our main interest was on the mean annual volumes of ground-water and overland runoff. Nevertheless, some limited success was achieved but it became apparent that results of equal suitability could be achieved by the use of simple and widely used methods for estimating runoff volume.

An overview of the water-balance procedures used to estimate the hydrologic parameters is presented in the following paragraph. This is followed by several subsections that discuss methods by which the variables and parameters in the water-balance equations can be estimated. Finally, the use of the procedures are demonstrated and an indication of the accuracy and reliability of the method is provided by application to an existing mine site.

Use of Eq. 1 requires that K, f_{sm} and f_{sn} be known. These parameters can be calculated directly from the long-term average quantities of annual direct and subsurface runoff. Subsurface runoff is estimated herein from a monthly subsurface water balance which, in turn, requires that long term average values of monthly direct runoff be estimated. Therefore, the first step is to estimate long term average values of monthly direct runoff. The second step is to compute infiltration from

$$I_i = P_i - Q_i \tag{6}$$

where I_i = average infiltrated volume per unit area for month i,

P_i = average precipitation available for direct runoff or infiltration in month i,

Q_i = average direct runoff in month i.

The infiltration values determined from Eq. 6 are input to the monthly soil-water balance:

$$W_i = I_i - E_{tai} - \Delta S_i \tag{7}$$

where W_i = volume of water per unit area passing below the root zone in month i,

E_{tai} = volume per unit area consumptively used from the root zone in month i (i.e., actual evapotranspiration),

ΔS_i = change in volume of water stored in the root zone per unit area in month i.

The summation of Q_i is the average annual direct runoff. Under the assumption that net changes in storage are zero for average annual conditions, the summation of W_i is the average annual subsurface runoff. A large time lag may exist between the time a particular W_i occurs and the time at which the water appears as runoff. However, the timing of subsurface runoff is not necessary for the application of this model. The above computations, performed for both the disturbed and undisturbed lands, permit calculation of K, f_{sm}, and f_{sn} directly from their definitions.

The SCS Method for Direct Runoff

A widely used method for computing the volume of direct runoff is the SCS curve number method described in detail in "Hydrology", Supplement A to Section 4 of the SCS National

Engineering Handbook and summarized elsewhere (e.g., Viessman, et al., 1977; Hahn and Barfield, 1978). The SCS method has been used to estimate direct runoff from daily precipitation data and from individual storms. Daily precipitation values are used in this report. The volume of direct runoff per unit area from a 24 hr precipitation value, P, is given by

$$P = \frac{(P - 0.2S)^2}{P + 0.8S} \tag{8}$$

where Q = direct runoff, volume per unit area (e.g., cm),

P = precipitation, volume per unit area (e.g., cm),

S = potential maximum retention, volume per unit area (e.g., cm).

The parameter S is related to a curve number CN by

$$S = \frac{1000}{CN} - 10. \tag{9}$$

The reader is cautioned that Eq. 9 gives S in inches, and must be converted to the appropriate unit if other than inches are being used. The curve number CN is in turn related to such factors as the hydrologic classification of the soil, the hydrologic condition of the watershed, the type and quantity of vegetal cover and the antecedent moisture conditions. In the interest of completeness, the procedures by which values of CN are determined are included in this report, even though they are readily available elsewhere.

The curve number for an average antecedent moisture condition (i.e., AMC II) is obtained from Table 1. The hydrologic soil group designation refers to the following conditions:

A) Soils exhibiting high infiltration rates, even when thoroughly wetted; deep, well drained, sands and gravels. Low runoff potential.

B) Soils exhibiting moderate infiltration rates when thoroughly wetted; moderately well to well-drained soils; medium to moderately coarse textured soils. Moderate runoff potential.

C) Soils exhibiting low infiltration rates when thoroughly wetted; restricted drainage by impeding layer or low permeability; medium to fine textured soils. Moderate to high runoff potential.

D) Soils exhibiting very low infiltration rates; soils with swelling clays, clay pan near the surface, water table near the surface, shallow soils over nearly impervious surface. High runoff potential.

A large number of soils throughout the United States have been given hydrologic classification. Also, detailed soils investigations are required as part of mining permit applications in most states, and data from the soils study should be used to assist in determining the hydrologic classification.

The curve number applicable for a particular runoff event depends upon the antecedent moisture condition as well as the factors listed in Table 1. The curve numbers corresponding to antecedent moisture conditions other than the average value of II can be determined from Table 2. The classification of antecedent moisture conditions is given below.

AMC I - Soils are dry but not to the wilting point

AMC II - Average soil moisture condition

AMC III- Soil is wet, nearly saturated from previous precipitation

Table 1

RUNOFF CURVE NUMBERS FOR AMC II

Land Use Or Cover	Treatment Or Practice	Hydrologic Condition	Hydrologic Soil Group A	B	C	D
Fallow	Straight Row	----	77	86	91	94
Row Crops	Straight Row	poor	72	81	88	91
		good	67	78	85	89
	Contoured	poor	70	79	84	88
		good	65	75	82	86
	Contoured & Terraced	poor	66	74	80	82
		good	62	71	78	81
Small Grain	Straight Row	poor	65	76	84	88
		good	63	75	83	87
	Contoured	poor	63	74	82	85
		good	61	73	81	84

(continued)

TABLE 1 (continued)

Land Use Or Cover	Treatment Or Practice	Hydrologic Condition	Hydrologic Soil Group A	B	C	D
	Contoured & Terraced	poor	61	72	79	82
		good	59	70	78	81
Legumes, Rotation Meadow	Straight Row	poor	66	77	85	89
		good	58	72	81	85
	Contoured	poor	64	75	83	85
		good	55	69	78	83
	Contoured & Terraced	poor	63	73	80	83
		good	51	67	76	80
Pasture,range		poor	68	79	86	89
		fair	49	69	79	84
		good	39	61	74	80
	Contoured	poor	47	67	81	88
		fair	25	59	75	83
		good	6	35	70	79
Meadow		good	30	58	71	78
Woods		poor	45	66	77	83
		fair	36	60	73	79
		good	25	55	70	77
Roads (including right-of-way)	Dirt		72	82	87	89
	Hard Surface		74	84	90	92

Table 2

CURVE NUMBERS FOR WET AND DRY ANTECEDENT CONDITIONS

CN for AMC II	CN for AMC I	CN for AMC III
100	100	100

(continued)

TABLE 2 (continued)

CN for AMC II	CN for AMC I	CN for AMC III
95	87	98
90	78	96
85	70	94
80	63	91
75	57	88
70	51	85
65	45	82
60	40	78
55	35	74
50	31	70
45	26	65
40	22	60
35	18	55
30	15	50
25	12	43
20	9	37
15	6	30
10	4	22
5	2	13

As noted previously, the long term average values of monthly and annual direct runoff are required. A straight forward way to estimate these quantities is to compute direct runoff from Eq. 8 for each day for which precipitation exceeded 0.2S as determined from the historical precipitation record. The individual values of direct runoff are summed by month and divided by the number of years of record used to determine the average direct runoff by month. This is a laborious computation to be made by hand, especially for situations in which daily precipitation frequently exceeds 0.2S. However, the computation can be made readily by almost any desk top computer. The salient features of an algorithm by which the average quantities of annual and monthly direct runoff can be computed are given below.

Before beginning the computation, the values of S corresponding to antecedent moisture conditions I, II, and III are determined. It is also necessary to quantify the conditions in which AMC I, II, and III are expected. For example:

A) AMC I applies if there has been zero precipitation for x days prior to the day in question,

B) AMC II applies if total precipitation in x days prior to the day in question is greater than zero but less than y.

C) AMC III applies if total precipitation in x days prior to the day in question is equal to or greater than y.

Specific values for x and y must be determined on a site-by-site basis using the best judgement of the user. With known values of S corresponding to each antecedent moisture condition and an unambiguous method by which the antecedent moisture condition is determined, the direct runoff can be calculated.

1) Determine if precipitation occurred on day j. If not, proceed to day j + 1. Otherwise,

2) determine applicable antecedent moisture condition and determine if precipitation on day j is greater than the corresponding value of 0.25. If not, proceed to day j + 1. Otherwise,

3) compute direct runoff from Eq. 8.

4) Go to day j + 1.

A less satisfactory method for estimating the average values of annual and direct runoff that does not require a computation of direct runoff for each day can be used when the above procedure is deemed impractical for some reason. The first step is to determine the value of S corresponding to AMC II. Intervals of daily precipitation are selected and a histogram of daily precipitation for each month is prepared from the historical record. Only precipitation values greater than 0.2S need be considered. The average direct runoff for month j is computed from

$$Q_j = \sum_{i=1}^{m} N_{ij} \frac{(P_{i,j} - 0.2S)^2}{P_{i,j} + 0.85} \qquad (10)$$

where Q_j = average direct runoff for month j

P_{ij} = midrange precipitation for histogram interval i in month j

$N_{i,j}$ = mean number of daily precipitation values in interval i for month j

m = number of histogram intervals

Eq. 10 also applies for calculation of average annual direct runoff by simply dropping the index j. The use of Eq. 10 on a monthly basis is demonstrated subsequently in the application to an existing mine.

Questions concerning the degree to which Eq. 10 agrees with calculation of direct runoff from each daily precipitation value in the historical record were investigated. This was accomplished by calculating average annual direct runoff by Eq. 10 and by individual days using 10 years of precipitation record from a humid region. Table 3 contains the histogram data and the calculation using Eq. 10. The value of S used in the calculation was 10.16 cm. Daily precipitation values greater than 0.2S = 2.03 cm result in direct runoff, therefore. In the ten years of record, there were a total of 185 days on which precipitation was greater than 2.03. During the ten years of record, there were 104 days on which the precipitation was between 2.03 and 3.30 cm, 28 days on which the precipitation was between 3.30 and 4.57 cm and so on. It will be noted that the precipitation interval used in Table 3 is 1.27 cm (0.5 inches). For each value of P_i in the second column, a value of direct runoff was computed and multiplied by the corresponding value of N to obtain the values in right-hand column. The sum of values in the right-hand column is average annual direct runoff and is 9.1 cm for this example.

TABLE 3

CALCULATION OF AVERAGE ANNUAL DIRECT RUNOFF FROM HISTOGRAM OF DAILY PRECIPITATION - PRECIPITATION INTERVAL = 1.27 cm

Histogram Interval cm	P_i cm	Number of Days In 10 Yr Record	Average Number Of Days Per Year, N_i	Q_i cm
2.03 - 3.30	2.67	104	10.4	0.4
3.30 - 4.57	3.94	28	2.8	0.9
4.57 - 5.84	5.21	23	2.3	1.8
5.84 - 7.11	6.48	18	1.8	2.5
7.11 - 8.38	7.75	8	0.8	1.7
8.38 - 9.65	9.02	1	0.1	0.3
9.65 - 10.92	10.29	0	0	0
10.92 - 12.19	11.56	2	0.2	0.9
12.19 - 13.46	12.83	0	0	0
13.46 - 14.73	14.10	1	0.1	0.6
		185	18.5	9.1

The direct runoff for each of the 185 days for which P > 2.03 cm was calculated and summed. The average annual direct runoff by this method is 8.7 cm. To obtain this estimate required 185 individual calculations using Eq. 8. The estimate of 9.1 cm was obtained by application of Eq. 8 only 10 times. To investigate the effect of the size of the precipitation interval, the histogram method was again used, but with an interval of 0.51 cm. The histogram method using an interval of 0.51 cm

resulted in average annual direct runoff of 8.8 cm which is nearly identical to that obtained using an interval of 1.27 cm.

It is concluded that calculation of direct runoff by summing the daily direct runoff values is preferred over the histogram method. The histogram method will greatly reduce the number of calculations required and will yield estimates that are within the confidence interval associated with the SCS direct runoff formula. However, the histogram method is less flexible with respect to adjustment of S in accordance with the antecedent moisture conditions that prevail.

Direct Runoff From Snowmelt

The procedure discussed above is applicable to direct runoff from precipitation in the form of rainfall. Snowfall contributes significantly to the total annual precipitation in many surface coal mining regions and must be accounted for in the water balance computations. Water that becomes available for direct runoff or infiltration as the result of snowmelt usually does so much more slowly than from precipitation. Seasonal runoff hydrographs resulting from the melting of snowpacks do not exhibit the sharp peaks and short durations that are characteristic of rainfall-runoff hydrographs. This is because the direct runoff contribution to the seasonal snowmelt hydrograph is usually small relative to the subsurface contribution.

For the purposes of the application intended in this report, the time and space distribution of snowmelt is of interest only insofar as it affects direct runoff. It is assumed that no direct runoff from snowmelt occurs if the daily rate of snowmelt is less than the base intake rate of the soil (i.e., less than the rate of infiltration after prolonged wetting). In those cases in which the rate of snowmelt exceeds the base intake rate, the volume of direct runoff is calculated by multiplying the difference between the rate of snowmelt and the intake rate by the time interval over which the rate of snowmelt exceeds the intake rate. In many cases the most important influence of snowmelt will be to cause the antecedent moisture condition to be AMC III, thus causing increased direct runoff from rainfall that may occur.

Basin-wide snowmelt rates are difficult to estimate. A rather crude method, but one for which the required data is almost always available, is the degree day index method. The equation is

$$M = k\,D \qquad (11)$$

where M = cm of melt over the snow covered area

D = degree days with 0°C as base

k = degree day factor

Table 4 contains values for k adapted from the U.S. Corps of Engineers (1956). The degree day factors in Table 4 are average values obtained from several years of record. The values are appropriate for use with the mean monthly temperature. Other investigators have found the degree day factor to range from 0.23 to 0.69 cm per degree day based on a mean temperature above 0°C (Horton, 1945, Gartska, et al., 1958). The melt rates are greatest in open area and least in densely forested areas.

TABLE 4

VALUES OF DEGREE DAY FACTOR*

Basin**	Forested Area %	Degree Day Factor April	May
CSSL	40	0.407	0.457
UCSL	90	0.169	0.329
WBSL	93	0.178	0.192

* cm of melt over the snow covered area per degree day above 0°C

** designates Central Sierra, Upper Columbia, and Willamette Basin Snow Laboratories, respectively

Computations of snowmelt in this application are made by determining the water equivalent in the snow pack at the beginning of the snowmelt season. This water equivalent is reduced in time by subtracting the volume of snowmelt from the remaining water equivalent. An average loss of 1.2 cm/mo (or more) water equivalent due to evaporation/sublimation may occur during the winter months (Gartska, 1964). Losses greater than 1.2 cm/mo are common under windy conditions. Evaporation/sublimation losses may significantly reduce the water equivalent at the beginning of snowmelt from the total accumulated snow precipitation.

A Surface Water Balance

A surface water balance on a monthly basis is used to compute the volume of water per unit area infiltrated into the soil. Evapotranspiration and subsurface runoff are components of the soil-water balance and are addressed in the next subsection. The surface water balance is expressed by

$$I_i = P_i - Q_i \quad (12)$$

where I_i = infiltrated volume per unit area in month i,

P_i = precipitation available for runoff or infiltration in month i and includes snowmelt,

Q_i = direct runoff in month i.

Equation 12 is intended to apply in the warm season months when water becomes available for infiltration or runoff by rainfall and snowmelt.

A Soil Water Balance

The volume of infiltrated water computed from the surface water balance is used as input to the root zone. Again, a monthly balance is used and is expressed by

$$W_i = I_i - E_{tai} - \Delta S_i \qquad (13)$$

where W_i = volume of water per unit area passing below the root zone in month i,

E_{tai} = volume per unit area consumptively used from the root zone in month i (i.e. actual evapotranspiration),

ΔS = change in volume of stored water in the root zone per unit area during month i.

There is a maximum value of the volume per unit area that can be stored in the root zone for prolonged periods. This volume, called the available water capacity (AWC), is equal to the difference in volumetric water content at field capacity and that at permanent wilting, multiplied by the effective depth of the root zone (McWhorter and Sunada, 1977). It is assumed that water in excess of that required to make the available water (AW) equal to AWC and to satisfy the demands of evapotranspiration in any particular month will pass through the root zone. Therefore, application of Eq. 13 requires an accounting of the available water because ΔS can never be larger than AWC-AW for a given month. For example, suppose that at the beginning of a particular month, the AW = 5 cm and that during the month I = 10 cm and E_{ta} = 3 cm. Therefore, during the month there has been a net input of 7 cm to the root zone. Part of this net input of 7 cm will go toward satisfying any deficit in AW and, if there remains any left over, that part will pass below the root zone and eventually become ground-water runoff. Suppose that AWC = 10 cm, then 5 of the 7 cm of net input will go into storage to make AW = AWC and 2 cm will become ground-water runoff. On the other hand, if AWC = 14 cm, the entire net input would go into storage bringing the AW to 12 cm and there would occur no ground-water runoff.

It should be pointed out that use of the term ground-water runoff for W is not meant to imply that this quantity of water will appear as watershed drainage in the month for which it is computed. Appearance of W as drainage may lag the time it occurs substantially. However, under the assumption that the system is in dynamic equilibrium (i.e. there is no net change in water storage during the year) all of the water represented by W will eventually drain from the watershed.

Use of Eq. 11 requires that E_{ta} be known. Wymore (1974) has made adjustments in the original Jensen-Haise (Jensen and Haise, 1963) equation that may be conveniently used for computing the potential evapotranspiration E_{tp}. The evapotranspiration under conditions of plentiful water E_t is obtained by multiplying E_{tp} by a water-use coefficient k_c. The Jensen-Haise equation as modified by Wymore (1974) is

$$E_{tp} = [0.25T - 0.123 + 0.13E_\ell]R_s \qquad (14)$$

where E_{tp} = potential evapotranspiration (cm)

T = temperature in degrees centigrade

E_ℓ = elevation in thousands of meters

R_s = evaporation equivalent of solar radiation (cm).

Application of Eq. 14 on a monthly basis requires the mean monthly temperature T and the total cumulative evaporation equivalent of solar radiation. The solar radiation is normally measured in calories per cm^2 per unit of time. The unit of time of interest in this application is the month. The evaporation equivalent of the solar radiation is obtained by dividing the solar radiation by the latent heat of vaporization of water (585 calories/cm^3).

Water is not always consumed at the potential rate, even when available water is not limiting. The evapotranspiration in the case when water is not limiting is given

$$E_t = K_c E_{tp} \qquad (15)$$

where K_c is a water-use coefficient that depends upon the vegetal cover and time of the year. Wymore (1974) provides the values of K_c shown in Table 5. Wymore assumed that phreatophytes would use water at the potential rate.

TABLE 5

WATER USE COEFFICIENTS FOR SELECTED COVERS

Type of Cover	K_c						
	Apr	May	Jun	Jul	Aug	Sep	Oct
Sagebrush-grass	0.60	0.80	0.80	0.80	0.71	0.53	0.50
Pinyon-Juniper	0.70	0.80	0.80	0.80	0.80	0.69	0.65
Mixed Mt. Shrubs	0.67	0.81	0.85	0.82	0.74	0.65	0.60
Conifer forest	0.71	0.80	0.80	0.80	0.79	0.75	0.71
Aspen forest	0.67	0.85	0.90	0.86	0.75	0.65	0.60
Rockland & misc.	0.60	0.65	0.65	0.65	0.60	0.50	0.50
Phreatophytes	1.00	1.00	1.00	1.00	1.00	1.00	1.00

(After Wymore, 1974)

Under most arid and semi-arid conditions, the cumulative evapotranspiration for the season as computed by summing the monthly values from Eq. 15 would exceed the total precipitation. This is because Eq. 15 is based on the assumption that water available is not limiting. Procedures have been presented (Jensen, et al., 1970; Hall, et al., 1979) to modify the value of K_c to include the effects of depletion of stored water. For the purposes of this report in which monthly values are used, it is simply assumed that, if E_t as calculated from Eq. 15 is larger than the sum of the AW and I for a particular month, then the actual evapotranspiration E_{ta} is equal to AW + I. When AW + I is greater than E_t, then $E_{ta} = E_t$.

Readers may find methods for the computation of evapotranspiration other than that above to be more convenient because of experience or data availability. The publication entitled Consumptive Use of Water and Irrigation Water Requirements (Jensen, 1973) presents a thorough and comparative discussion of many different methods.

SUMMARY

The above paragraphs describe a method by which the mean annual precipitation is partitioned into direct runoff and ground-water runoff by months. The annual direct and ground-water runoff quantities are computed by summing the monthly values. It is a simple matter to compute K, f_{sm}, and f_{sn} directly from their definitions and the quantities of runoff in each component. The calculation is best summarized by the detailed example computation that follows.

Before proceeding to the example, it is convenient to summarize the data requirements in the following list.

1) Historical record of daily precipitation

2) Mean monthly temperatures

3) Mean monthly solar radiation

4) Elevation

5) Soil data to permit hydrologic classification

6) Vegetal type and cover

7) Land use

8) Base intake rate of soils

9) Available water capacity and rooting depth

10) Estimates of P_{gm} and P_{sm}

11) Estimates of P_n from pre-mining monitoring

12) Fraction of watershed area to be mined

EXAMPLE

The example calculation presented in this subsection is intended to provide both a demonstration of the computational procedure and an indication of the accuracy that can be expected from the method. It is important to understand that the purpose of the model is to predict the dissolved-solids concentration in the combined direct and subsurface runoff from a watershed that has been partially disturbed by mining. The predicted concentration is that which is expected on a long-term average basis. Comparison of the predicted value of P_t with that measured over a short period of record provides only an indication of the reliability. A long-term record (perhaps 20 yrs or more) is required for a thoroughly valid test. The method recommended in the previous subsections of this report for calculating the hydrologic parameters involves estimating such individual quantities as direct runoff and combined direct and subsurface runoff. If should be recognized that considerable error in the estimation of these individual quantities can be tolerated, provided that the required ratios are satisfactorily accurate. Certainly if both the required ratios and the individual quantities defining the ratios are accurately predicted, increased confidence in the procedures is warranted.

The method recommended in the foregoing portions of this report are applied to a partially mined watershed comprising part of the Edna Mine in northwest Colorado. This site was selected for the example application primarily because measurements of P_t have been made previously and are available for comparison with the value of P_t predicted herein. At the time the field studies were conducted at the Edna Mine, the method provided in this report for computation of the hydrologic parameters was not contemplated. Therefore, all of the data required by the method was not obtained, at least in the detail that is desirable. For this reason the example application provides a demonstration of some of the judgements that may be required in other applications. Data form the Edna Mine field study include both measured values for P_t and measured total runoff. Therefore, it is possible to compare predicted values of both P_t and total runoff to measured values. The latter comparison will provide an indication of the degree of reliability of the water balance computations.

Description of Site

Descriptions of the Edna Mine site are detailed elsewhere (McWhorter, et al., 1979; Rowe and McWhorter, 1978; McWhorter, et al., 1975) and only the features salient to the present application are repeated here. The portion of the watershed in which the Edna Mine is located ranges in elevation from 2195 m to 2500 m above mean sea level. The mined portion of area has not been regraded or top-soiled at the time of the above referenced field studies. Vegetation on the spoil was sparse relative to that on the undisturbed ground and consisted of short grasses and weeds. Vegetation on the undisturbed portion was a mixture of native grasses, oak brush, sage brush, and some aspen and conifers on the highest portions of the area.

Other characteristics of the site are given in the following paragraphs where they relate to a particular computation.

1. The first step in the computation is to assemble the climatological data that is required. It is recommended that no less than 20 years of record be used. In this example compuation the precipitation record for Pyramid, Colorado is used for precipitation on the mine site. The Pyramid station is at elevation 2465 m and located only a few miles from the study area and in the same watershed. Unfortunately, there exists no temperature record; either on the mine site or at Pyramid. Temperature records at Steamboat Springs, Colorado (elevation 2034 m) were utilized and adjusted to the mean elevation of the mine site (2350 m) by decreasing the temperatures at the rate of 0.55°C per 100 m of elevation. No

solar radiation data were available for the mine site and data from Fort Collins, Colorado were used. Fort Collins is nearly the same latitude as the mine site. The long-term mean climatological data that are estimated to apply at the mine site are given in Table 6. Also shown in Table 6 are values of potential evapotranspiration, E_{tp}. These were calculated from Eq. 14 using the mean temperatures and R_s values as shown. In addition to the data in Table 6, the daily precipitation values are required for calculation of direct runoff. These data are presented in the next step.

TABLE 6

CLIMATOLOGICAL CHARACTERISTICS FOR MINE SITE

Month	Total P cm	Snow Prec cm	Rain Prec cm	Aver Temp °C	R_s cm	E_{tp} cm
Jan	4.6	4.6	0	-11.8	12.1	---
Feb	4.5	4.5	0	- 9.3	14.7	---
Mar	5.6	5.6	0	- 4.9	23.3	---
Apr	5.0	3.7	1.3	1.5	25.0	5.4
May	3.6	0.8	2.8	6.8	28.3	10.0
Jun	3.8	0	3.8	10.7	33.0	14.8
Jul	3.3	0	3.3	14.4	30.3	16.4
Aug	4.3	0	4.3	13.5	27.1	14.1
Sep	4.4	0.6	3.8	9.1	23.4	9.6
Oct	4.2	1.3	2.9	3.9	19.1	5.3
Nov	4.0	3.0	1.0	- 4.0	11.8	---
Dec	5.1	5.1	0	- 9.9	10.5	---
	52.4	29.2	23.2		258.6	75.6

II. The second step is the estimation of direct runoff from precipitation in the form of rainfall. Determination of the applicable curve numbers is required and is accomplished from Tables 1 and 2, together with knowledge of the cover and hydrologic classification of the soil. The mined area in the present example had not been topsoiled or revegetated at the time this computation applies. Weathering of the fragmented shale and sandstone spoil on the surface had tended to produce a layer of fairly low permeability and a moderately high runoff potential. The hydrologic soil group classification believed to best represent the spoil is C. Also, the spoil is range land judged to be in good to fair hydrologic condition. From Table 1, the curve number for the spoil was estimated to be 80 for AMC II.

The undisturbed ground has a high infiltration rate (McWhorter, et al., 1975) but is only moderately well drained. The hydrologic soil group classification was judged to be between A and B. For range land in fair hydrologic condition it was estimated that CN = 62 for the undisturbed ground at AMC II.

Snowmelt occurs mainly in April on the mine site and it was estimated that AMC III is the prevailing antecedent moisture condition in April. It was also judged that AMC II was appropriate for May and that AMC I applied during the remainder of the summer. Table 2 was used to determine the curve numbers for AMC I and AMC III for both the mined and unmined portions, and the corresponding values for S were calculated from Eq. 9. All of these data are summarized in Table 7.

TABLE 7

SUMMARY OF PARAMETERS FOR CALCULATION OF DIRECT RUNOFF

Month	Spoil			Natural		
	AMC	CN	S(cm)	AMC	CN	S(cm)
April	III	90	2.54	III	80	6.35
May	II	80	6.35	II	62	15.21
June - Oct	I	62	15.20	I	43	33.78

It was noted from Eq. 8 that direct runoff occurs for P greater than 0.2S. In April direct runoff will occur from the spoil for P greater than 0.51 cm, and for P greater than 1.27 cm in May. Similarily, the threshold value of rainfall can be computed for each period for both the spoil and the natural ground. In this example the histogram method was used to compute the direct runoff. Table 8 shows the computation for the spoil. The historical record of rainfall was examined to determine the number of days that rainfall amounts were in the intervals 0.51 - 1.27 cm, 1.27 - 2.03 cm and etc. for each month. The 20 years of record showed that there were 54 days in April for which rainfall was between 0.51 cm and 1.27 cm and 9 days for which rainfall was between 1.27 and 2.03 cm, for example. There were only 2 days in the 20 year record for which rainfall was greater than 2.79 cm, and direct runoff for these two days was computed directly from Eq. 8. There were no days in June, July, or August for which rainfall exceeded the threshold of 3.05. In September

and October there were a total of three days and direct runoff was computed for each individual day as shown in the table. The same calculations for the natural ground are shown in Table 9.

TABLE 8

CALCULATION OF DIRECT RUNOFF FROM RAINFALL
DISTURBED GROUND (SPOIL)

Month	Histogram Interval cm	P_i cm	Number of Days in 20 yr Record	Avg No of Days per Year, N_i	Q_i* cm
April	0.51 - 1.27	0.89	54	2.7	0.13
	1.27 - 2.03	1.65	9	0.45	0.16
	2.03 - 2.79	2.41	4	0.20	0.16
	2.95	2.95	1	0.05	0.06
	3.18	3.18	1	0.05	0.07
					0.58
May	1.27 - 2.03	1.65	9	0.45	0.01
	3.35	3.35	1	0.05	0.03
					0.04
June	>3.05	----	0	0	0
July	>3.05	----	0	0	0
Aug	>3.05	----	0	0	0
Sept	7.37	7.37	1	0.05	0.05
Oct	3.30	3.30	1	0.05	~0
	3.81	3.81	1	0.05	~0
					0.67

* April, $Q_i = \frac{(P_i - 0.51)^2}{P_i + 2.03}$; May, $Q_i = \frac{(P_i - 1.27)^2}{P_i + 5.08}$

June through Oct, $Q_i = \frac{(P_i - 3.05)^2}{P_i + 11.93}$

TABLE 9

CALCULATION OF DIRECT RUNOFF FROM RAINFALL
NATURAL GROUND

Month	Histogram Interval cm	P_i cm	Number of Days in 20 yr Record	Avg No of Days per Year, N_i	Q_i* cm
April	1.27 - 2.03	1.65	9	0.45	
	2.03 - 2.79	2.41	4	0.20	0.16
	2.95	2.95	1	0.05	0.06
	3.18	3.18	1	0.05	0.07
					0.29
May	3.35	3.35	1	0.05	0.03
June	>6.86	----	0	0	0
July	>6.86	----	0	0	0
Aug	>6.86	----	0	0	0
Sept	7.37	7.37	1	0.05	~0
Oct	>6.86	----	0	0	0
					0.32

* April, $Q_i = \frac{(P_i - 1.27)^2}{P_i + 2.03}$; May, $Q_i = \frac{(P_i - 3.05)^2}{P_i + 11.93}$

June through Oct, $Q_i = \frac{(P_i - 6.76)^2}{P_i + 27.03}$

III. The third step in the computation is estimation of direct runoff from snowmelt. In this example, it is estimated that sublimation/evaporation losses are 1.5 cm per month for each month that continuous snow cover exists (i.e., November through April). Total precipitation in the form of snowfall for these 6 months is 26.5 cm (Table 6). The loss is 9.0 cm leaving an estimate water equivalent of 17.5 cm to become available for direct runoff or infiltration during the spring snowmelt season.

The degree day factor applicable to this site is not known. Using the largest value of k for April from Table 4 and a mean April temperature of 1.5°C from

Table 6 yields an average daily melt rate of 0.6 cm/d. At this average rate, the 17.5 cm of water equivalent would require 29 days to melt. Experience at the mine site suggests that snowmelt is completed within the first 10 days or so of May. This indicates that a melt rate of 0.6 cm/d is probably somewhat too large. Even so, the calculated melt rate is believed to be substantially less than the base intake rate of the spoil and is much less than the base intake rate of the natural ground, Therefore, it is assumed that the entire 17.5 cm of water equivalent melts in April and that no direct runoff is produced from the melt, either on the spoil or natural ground.

IV. The fourth step in the computation is the surface water balance calculation from which monthly values of infiltration are estimated. This is accomplished using Eq. 12. The computation for the Edna Mine example is shown in Table 10. The calculation shows that a total of 42.7 cm infiltrate the spoil and 43.1 cm infiltrate the natural soil.

TABLE 10

SUMMARY OF SURFACE WATER BALANCE

Month	Total Prec	Loss From Snow Pack	Available Prec		Direct Runoff		Infiltration	
			Snow	Rain	Spoil	Natural	Spoil	Natural
	cm	cm	cm	cm	cm	cm	cm	cm
Jan	4.6	1.5	0	0	0	0	0	0
Feb	4.5	1.5	0	0	0	0	0	0
Mar	5.6	1.5	0	0	0	0	0	0
Apr	5.0	1.5	17.5	1.3	0.58	0.29	18.2	18.5
May	3.6	0	0.8	2.8	0.04	0.03	3.6	3.6
Jun	3.8	0	0	3.8	0	0	3.8	3.8
Jul	3.3	0	0	3.3	0	0	3.3	3.3
Aug	4.3	0	0	4.3	0	0	4.3	4.3
Sep	4.4	0	0.6	3.8	0.05	0	4.3	4.4
Oct	4.2	0	1.3	2.9	0	0	4.2	4.2
Nov	4.0	1.5	0	1.0	0	0	1.0	1.0
Dec	5.1	1.5	0	0	0	0	0	0
	52.4	9.0	20.2	23.2	0.7	0.3	42.7	43.1

V. The fifth step in the computation is the soil-water balance. In order to make the soil-water balance calculations it is necessary to estimate the available water capacity. In the case of the spoil, laboratory measurements indicate that the available water for the

spoil is about 0.11 cm/cm. It is estimated that the effective rooting depth is 60 cm yielding a value of AWC equal to 6.6 cm. The available water per centimeter for the soil is probably about the same as for the spoil, but the effective rooting depth is judged to be somewhat greater. The available water capacity for the soil is estimated at AWC = 8.0 cm.

In addition to AWC it is also necessary to determine the water-use coefficients that apply. Table 11 gives the values for K_c that are deemed most reasonable for the spoil and for the natural ground at the Edna Mine.

TABLE 11

WATER USE COEFFICIENTS FOR EXAMPLE

Month	K_c	
	Spoil	Natural
April	0.60	0.67
May	0.72	0.83
June	0.72	0.88
July	0.72	0.84
Aug	0.65	0.75
Sept	0.51	0.65
Oct	0.50	0.60

The values in Table 11 for spoil are average values of the Wymores water-use coefficients for rockland and sagebrush-grass, (Table 5). The values for the natural ground are average values from Table 5 for aspen forest and mixed mountain shrubs.

The calculations of the soil-water balance for the spoil and the natural ground are shown in Tables 12 and 13, respectively. It is necessary to guess at the quantity of available water in storage at the beginning of the month for which the calculation is initiated. The calculation for the entire season will not balance unless the initial guess is correct. If balance is not achieved, then a new guess is made and the computations repeated until a balance is achieved. In arid and semi-arid regions where the potential for water use is greater than the quantity available, it is usually possible to eliminate the trial and error. This is accomplished by initiating the calculation for a month that is preceeded by months of low precipitation and high potential

water use. In this case the best first estimate for the quantity of stored water is zero.

The computations shown in Tables 12 and 13 were initiated for the month of October because it appeared reasonable that the available water would be completely depleted by the beginning of October. Transfer of water to or from the root zone is assumed to be zero from December through March. Therefore, the first active month following November is April, followed by May and so on. The next step is to fill in the columns headed I (infiltration), E_{tp} (potential evapotranspiration) and E_t (evapotranspiration with water availability not limiting). In this example, values of I were given previously in Table 10, and values of E_{tp} were listed in Table 6. The values for E_t shown in Tables 12 and 13 were obtained by multiplying E_{tp} by the appropriate water-use coefficient (Table 11).

The column head AW is the available water in storage at the beginning of the month. The deficit column is simply the quantity of water required to bring the root zone to maximum storage capacity (i.e. AW + deficit = AWC). As explained above, the calculation is initiated by assuming a value for AW. A value of AW equal to zero was used in this example for October 1. Thus, the deficit on Oct. 1 is 6.6 cm for the spoil (Table 12). During October, 4.2 cm of infiltration occurs. In the same time period the evapotranspiration demand (E_t) was 2.6 cm. Because infiltration exceeded demand, the entire demand of 2.6 cm was satisfied and the actual evapotranspiration (E_{ta}) equalled E_t. The difference between infiltration and evapotranspiration went toward making up the deficit in soil-water storage. Thus, the change in storage (ΔS) was positive and equal to 1.6 cm. No percolation (W) occurred because the storage deficit was sufficient to accommodate the entire excess of infiltration over demand.

The 1.6 cm of water that went into storage in October causes AW to equal 1.6 cm at the beginning of November. The corresponding deficit is 5.0 cm. There is no evapotranspiration demand in November so the 1 cm of infiltration goes into storage, making the AW equal to 2.6 cm at the beginning of April. Infiltration in April is 18.2 cm and is more than sufficient to meet

the demand of 3.2 cm. Therefore, $E_{ta} = E_t = 3.2$ cm, leaving an excess of 15.0 cm. Four centimeters of this excess go to make up the remaining deficit. So $\Delta S = +4.0$. The remainder is 11.0 cm and becomes percolation below the root zone.

TABLE 12

SOIL-WATER BALANCE IN SPOIL*

Month	AW** cm	Deficit** cm	I cm	E_{tp} cm	E_t cm	E_{ta} cm	ΔS cm	W cm
Oct	0	6.6	4.2	5.3	2.6	2.6	+1.6	0
Nov	1.6	5.0	1.0	0	0	0	+1.0	0
Apr	2.6	4.0	18.2	5.4	3.2	3.2	+4.0	11.0
May	6.6	0	3.6	10.0	7.2	7.2	-3.6	0
Jun	3.0	3.6	3.8	14.8	10.7	6.8	-3.0	0
Jul	0	6.6	3.3	16.4	11.8	3.3	0	0
Aug	0	6.6	4.3	14.1	9.2	4.3	0	0
Sep	0	6.6	4.3	9.6	4.9	4.3	0	0
			42.7	75.6	49.6	31.7	0	11.0

* AWC = 6.6 cm
** Evaluated at beginning of month

TABLE 13

SOIL-WATER BALANCE IN NATURAL AREA

Month	AW** cm	Dificit** cm	I cm	E_{tp} cm	E_t cm	E_{ta} cm	ΔS cm	W cm
Oct	0	8.0	4.2	5.3	3.2	3.2	+1.0	0
Nov	1.0	7.0	1.0	0	0	0	+1.0	0
Apr	2.0	6.0	18.2	5.4	3.6	3.6	+6.0	8.6
May	8.0	0	3.6	10.0	8.3	8.3	-3.3	0
Jun	4.7	3.3	3.8	14.8	13.0	8.5	-4.7	0
Jul	0	8.0	3.3	16.4	13.8	3.3	0	0
Aug	0	8.0	4.3	14.1	10.6	4.3	0	0
Sep	0	8.0	4.3	9.6	6.2	4.3	0	0
			42.7	75.6	58.7	35.5	0	8.6

May is begun with a 6.6 cm of water in storage. The evapotranspiration demand for May is 7.2 cm. This demand is greater than the infiltration of 3.6 cm but the sum of infiltration and available water is sufficient to meet the demand. Thus, $E_{ta} = E_t = 7.2$ cm. To meet this demand, 3.6 cm of stored water were required, so $\Delta S = -3.6$ cm and no percolation occurrs.

The available water at the beginning of June is 3.0 cm since 3.6 cm of stored water were used in May. The water-use demand in June is 10.7 cm, a quantity that exceeds the sum of AW and I. Therefore, the actual evapotranspiration is limited to $AW + I = 6.8$ cm and all remain stored water is used (i.e. $\Delta S = -3.0$ cm). For the remainder of the season, the evapotranspiration demand exceeds the infiltration so $E_{ta} = I$ and AW remains equal to zero through September. The guess that $AW = 0$ on October 1 was correct and no new trial is required.

The water balance computations are now complete. The procedure predicts that average annual direct runoff from the spoil is 0.7 cm and subsurface runoff is 11.0 cm for a total runoff value of 11.7 cm. The corresponding values on the natural ground are 0.3 cm, 8.6 cm, and 8.9 cm.

VI. Once the water balance calculations are complete, it is a simple matter to calculate f_{sn}, f_{sm} and K, directly from their definitions:

$$f_{sn} = \frac{Q}{Q+W} = \frac{0.3 \text{ cm}}{8.9 \text{ cm}} = 0.04, \text{ natural ground}$$

$$f_{sm} = \frac{Q}{Q+W} = \frac{0.7}{11.7} = 0.06, \text{ spoil}$$

$$K = \frac{\text{total runoff from natural ground}}{\text{total runoff from spoil}} = \frac{8.9}{11.7} = 0.76$$

VII. The final step is to determine the values of the chemical parameters P_{sm}, P_{gm}, P_{sn} and P_{gn}. It is anticipated that pre-mine monitoring will usually permit P_n to be estimated directly. In those cases, it is not necessary to estimate f_{sn}, P_{sn} or P_{gn}. Mining at the Edna Mine preceeded the field studies, however, and it is necessary to estimate the parameters for the natural ground as well as for the mined portion.

Field plots were established in the spoil at the Edna Mine from which both direct runoff and subsurface

runoff was collected (McWhorter, et al., 1979). Data from these plots suggested that $P_{sm} = 150$ mg/ℓ and $P_{gm} = 3030$ mg/ℓ. In the same study samples of surface and subsurface waters indicate that $P_{sn} = 150$ mg/ℓ and $P_{gn} = 460$ gm/ℓ.

Equations 2 and 3 are used to calculate P_m and P_n, respectively, using f_{sn} and f_{sm} as determined in Step VI. Finally, P_t is computed using the value of K from Step VI together with the appropriate value for R.

COMPARISON OF PREDICTED AND MEASURED VALUES

Table 14 summarizes the values of the model parameters as determined above for the Edna Mine. The field studies conducted at the Edna Mine resulted in measured values for P_t on 4 individual watersheds comprising a portion of the Edna Mine. Annual data for either 2 or 3 years were utilized to compute P_t. The fraction of each watershed mined was different and, therefore, resulted in a different value for R. The model parameters in Table 14 were used to calculate predicted values of P_t for each watershed. The predicted and measured values are presented in Table 15. As explained by McWhorter, et al., (1979) some interchange of water between watersheds C9 and C10 was suspected. For this reason, data from the combined watershed C9 + C10 is also presented. It is believed that the degree of agreement between the predicted and measured values is satisfactory. The fact that the predicted values are above the measured values in some cases and below in others is noteworthy.

Further insight into the reliability of the computational procedure is provided by comparing the predicted values of total runoff with the measured values. Adequate total runoff measurements are available for watersheds C3, C9 and C10. Together, these three watersheds total $8.06 \times 10^6 m^2$. In calendar year 1975, the measure runoff per unit area was 8.7 cm. The fraction of this total area that consisted of spoil was 0.467, the remainder being undisturbed ground. The water balance calculations predict runoff of 8.9 cm on natural ground and 11.7 cm on spoil. Thus, the predicted runoff is

$$(0.467)(11.7) + (0.533)(8.9) = 10.2 \text{ cm.}$$

The predicted total runoff is 1.5 cm greater than the measured value for 1975. The 1975 precipitation was 2.8 cm less than the mean vlaue used in the prediction. This difference may provide a partial explanation for the difference between measured and predicted runoff. However, the confidence interval about the predicted value, while not known, is undoubtedly at

least as large as the difference between the measured and predicted values.

TABLE 14

SUMMARY OF PREDICTED MODEL PARAMETERS

Hydrologic	Chemical	Calculated
$f_{sn} = 0.04$	$P_{sm} = 150$ mg/ℓ	$P_m = 2860$ mg/ℓ; Eq. 2
$f_{sm} = 0.06$	$P_{gm} = 3030$ mg/ℓ	$P_n = 450$ mg/ℓ; Eq. 3
$K = 0.76$	$P_{sn} = 150$ mg/ℓ	
	$P_{gn} = 460$ mg/ℓ	

TABLE 15

COMPARISON OF PREDICTED P_t WITH MEASURED VALUES

Watershed No.	R	Predicted P_t mg/ℓ	Avg. Meas. P_t mg/ℓ	Range of Meas. P_t mg/ℓ
C 3	0.47	2220	1840	1610 - 2030
C 5	0	2860	2910	2830 - 3080
C 9	1.86	1450	1240	1190 - 1290
C10	1.27	1670	1850	1850 - 1860
C 9 + C10	1.44	1600	1550	1520 - 1580

SECTION 5

SINGLE-WELL AQUIFER TESTS IN COAL HYDROLOGY

Aquifer tests are the primary means of determining the hydraulic parameters of water-bearing strata that are required for projecting the effect of mining on the ground-water regime and for estimating the quantities of ground-water inflow that can be anticipated in the mine workings. Ideally, aquifer tests are performed by pumping at one location and observing the response in one or more observation wells. Experience in the Rocky Mountain and Northern Great Plains coal fields suggest that the water-bearing strata (including the coal seams) can be expected to exhibit low permeabilities. Aquifer tests in these materials require very low pumping rates over a long time period to insure that meaningful responses will be obtained in the observation wells. It is not unusual to find that pumping rates on the order of 1 to 5 liters/min are required to keep the pumped well from dewatering before meaningful responses are observed in even nearby observation wells. Control of such small pumping rates at a constant value is difficult and, even when successfully accomplished, analysis of the data is complicated by the effects of well-bore storage that are almost always important in such cases.

The above described difficulties suggest that many of the advantages that are attributed to multi-well aquifer tests relative to single-well tests under normal circumstances, often are not obtained in practice in the low-permeability materials encountered in coal hydrology studies. Furthermore, the additional drilling and completion work required for multi-well tests makes them more costly than single-well tests.

Single-well tests include a number of specific tests. Among them are: a) the slug test in which water is "instantaneously" added to or displaced from the well-bore and the return of the water level towards its equilibrium position is monitored, b) the pumping/recovery test in which the well is pumped for a period and the water level is monitored in the well during and after pumping, c) the pressure-pump-in method in which water is pumped into the well and discharge and pressure are monitored, and d) the auger-hole method which is very similar to the slug method. Of particular interest in this report are the slug test and the pumping/recovery test.

The standard theories and procedures for analyzing the slug test and the pumping/recovery test are discussed in the following sub-sections for the purposes of setting the stage for the development and understanding of a new procedure for analyzing the recovery under conditions where the standard methods are inappropriate.

RECOVERY FOLLOWING AN INSTANTANEOUS DRAWDOWN IN THE WELL-BORE

Methods of performing and analyzing slug tests are well developed and well-known, and no new insights to this test will be provided here. The main interest in the slug test for the purposes of this report derives from the fact that it represents a limiting case for the pumping/recovery test which is discussed in detail subsequently.

Slug tests are normally performed by adding to or displacing from the well-bore a quantity of water. The return of the water level toward its initial equilibrium position is monitored and this recovery is analyzed by fitting a theoretical response function to the data. The theoretical response functions are solutions to the differential equation of ground-water flow and are based upon the condition that the initial drawdown or buildup of water level in the well is created instantaneously. Solutions for the slug test have been derived by Hvorslov (1951), by Cooper et al. (1967), and by Bouwer and Rice (1976). These solutions differ somewhat because of different assumptions made concerning the importance of aquifer storage on the flow induced in the aquifer.

Of the three solutions referenced above, the most rigorously correct response function is that of Cooper et al. and is given by

$$\delta_w = \frac{8\delta_o\alpha}{\pi^2} \int_o^\infty \frac{\exp(-\beta u^2/\alpha)}{u\Delta u}\, du \qquad (1)$$

where δ_w = drawdown (or buildup) in the well

δ_o = initial drawdown (or buildup) in the well

$\alpha = Sr_w^2/r_c^2$

S = storage coefficient

r_w = radius of well

r_c = radius of casing

$\beta = Tt/r_c^2$

T	=	transmissivity
t	=	time
u	=	integration variable
Δu	=	a combination of Bessel functions

Cooper et al. provide a table of values of the integral which permit the solution to be displayed graphically in the form of a family of curves of s_w/s_o vs Tt/r_c^2 with α as the curve parameter. This solution rigorously includes the effects of well-bore storage as well as the effects of aquifer storage, and from this solution it is apparent that the particular flow under consideration here is insensitive to the aquifer storage coefficient in the range of values for S normally encountered in confined aquifers. This conclusion tends to justify the assumption implicit in the Hvorslov (1951) method that the distribution of piezometric head in the aquifer is that given by the appropriate solution to the Laplace equation which does not account for aquifer storage. It is not surprising, therefore, that the Cooper et al. and the Hvorslov methods usually yield results in close agreement with one another.

The characteristic of this test that is particularly important in subsequent developments in this report is that flow to the well is initiated by creating a drawdown at the well-bore at t = 0 by displacing water from the well-bore only. It is assumed that no water has been withdrawn from the aquifer at the beginning of recovery in this test.

RECOVERY FOLLOWING A PUMPING PERIOD - STANDARD THEORY

While the response functions for the slug test are based on the assumption that the drawdown in the well at the beginning of recovery is created without withdrawing any water from the aquifer, the standard theory for analyzing the recovery of water level following a pumping period assumes that all of the pumped water is derived from aquifer storage. Furthermore, the standard theory of recovery following a pumping period assumes that aquifer discharge is zero during recovery (i.e., well-bore storage is neglected) while the slug theories account explicitly for the discharge to the well-bore required for the recovery of water levels. Thus, the two theories represent opposing extremes.

The Theis (1935) solution for the drawdown in a well pumped at constant rate Q is

$$s_w = \frac{Q}{4\pi T} \int_{\beta}^{\infty} \frac{\exp(-\chi)}{\chi} \, d\chi \qquad (2)$$

where Q = pumping rate

$\beta = Sr_w^2/4Tt$

χ = dummy variable of integration

and other parameters are as defined previously. For values of β less than about 0.05, the integral can be closely approximated by the first two terms in a series expansion and

$$s_w(t) = \frac{Q}{4\pi T}\left\{\ln\frac{4Tt}{Sr_w^2} - 0.5772\right\} \quad (3)$$

The theory of superposition in the time domain (McWhorter and Sunada, 1977) is used to write

$$s_w(t) = \frac{Q}{4\pi T}\left\{\ln\frac{4Tt}{Sr_w^2} - 0.5772\right\} - \frac{Q}{4\pi T}\left\{\ln\frac{4T(t-t_o)}{Sr_w^2} - 0.5772\right\} \quad (4)$$

for the drawdown in the well after pumping ceases. The parameter t_o is the pumping period. A simplification results in

$$s_w(t) = \frac{Q}{4\pi T}\ln\frac{t}{t-t_o}, \quad t > t_o \quad (5)$$

which is widely used to determine the transmissivity T using the recovery data. Particularly important, in the context of this report, is the implicit assumption that aquifer discharge ceases immediately upon the cessation of pumping.

RECOVERY AFFECTED BY AFTERFLOW

The aquifer discharge to a well is not zero immediately following the cessation of pumping, because some discharge is required to refill the well-bore as the water level rises. Throughout the remainder of this report, the aquifer discharge to the well-bore after pumping ceases will be referred to as afterflow. There are, indeed, many situations in which the afterflow discharge is very small relative to the aquifer discharge during the pumping period and Eq. 5 will yield a valid estimate of transmissivity T. However, in cases where the aquifer transmissivity is small, the afterflow discharge can be of the same order as the aquifer discharge during pumping. The limiting case is that in which the aquifer discharge during pumping is zero and the only aquifer discharge is afterflow. This limiting case is represented by the slug test. The response function developed below accounts for the effects of afterflow of any relative importance and incorporates the

Cooper et al. (1967) slug test response and Eq. 5 as limiting conditions at the two extremes.

The drawdown in a homogeneous, isotropic aquifer caused by pumping a well at a constant rate is described by the solution to the following boundary value problem.

$$\frac{\partial^2 s}{\partial r^2} + \frac{1}{r}\frac{\partial s}{\partial r} = \frac{S}{T}\frac{\partial s}{\partial t}, \quad r \geq r_w \tag{6}$$

$$s(\infty,t) = 0 \tag{7}$$

$$s(r,o) = 0, \quad r \geq r_w \tag{8}$$

$$2\pi r_w T \frac{\partial s(r_w,t)}{\partial r} - \pi r_c^2 \frac{ds_w}{dt} = -Q \tag{9}$$

where r is the radial coordinate and other symbols are as previously defined. The boundary condition expressed in Eq. 9 ensures that the contributions to the pumping rate Q from both the aquifer and from well-bore storage are properly considered. The first term on the left is the aquifer discharge to the well-bore and the second term on the left is the discharge contributed by change in the volume of water standing in the well-bore. Papadopulos and Cooper (1967) give the equation for drawdown in the pumped well, derived from the solution to the above boundary value problem:

$$s_w(t) = \frac{Q}{4\pi T}\left\{\frac{32\alpha^2}{\pi^2}\int_o^\beta \frac{\left[1 - \exp\left(\frac{-\chi^2}{4u}\right)\right]}{\chi^3 \Delta\chi}\, d\chi\right\} \tag{10}$$

$$= \frac{Q}{4\pi T} F_p(\alpha,t) \tag{11}$$

where χ = integration variable

$u = Sr_w^2/4Tt$

$\Delta\chi$ = combination of Bessel functions

and F_p is the dimensionless function in braces. Equation 10 describes the drawdown as a function of time during the pumping period.

The drawdown during the recovery period after pumping ceases can be obtained using the theory of superposition and is

$$s_w(t) = \frac{Q}{4\pi T}\left\{F_p(\alpha,t) - F_r(\alpha,t)\right\}, \quad t \geq t_o \tag{12}$$

where

$$F_p = \frac{32\alpha^2}{\pi^2}\int_o^\infty \frac{\left[1 - \exp\left(-\left(\frac{4Tt\chi^2}{\alpha r_c^2}\right)\right)\right]}{\chi^3\Delta\chi}\, d\chi \tag{13}$$

and

$$F_r = \frac{32\alpha^2}{\pi^2}\int_o^\infty \frac{\left[1 - \exp\left\{-\frac{4T(t-t_o)\chi^2}{\alpha r_c^2}\right\}\right]}{\chi^3\Delta\chi}\, d\chi \tag{14}$$

The solution represented in Eq. 12 gives the drawdown during the recovery period and gives full consideration to the relative effects of afterflow. Furthermore it is an exact solution entailing no approximations other than those inherent in the differential equation (Eq. 6). It is, of course, necessary to evaluate the integral functions by numerical integration and to provide the solution in graphical and/or tabular form. This was accomplished and the results are presented in Table 16 in a form consistent with the format used by Cooper et al. (1967) to present their solution for the slug test case. Explicitly, the ratio of the drawdown at a particular time to the drawdown s_o at the beginning of recovery is expressed as a function of the dimensionless recovery time Tt_r/r_c^2 for different dimensionless pumping times, Tt_o/r_c^2. The response functions are shown graphically in Figure 1. The curve in Figure 1 corresponding to a pumping period of zero duration is the slug test response. It is clear that the slug test response is a natural limiting member of the family of curves representing the recovery following different pumping periods. It will be noted that the response functions in Table 16 and shown graphically in Figure 1 correspond to a value of α equal to 10^{-4}. A separate table and family of curves must be prepared for each value of α, strictly speaking. However the response functions are much less sensitive to values of α than to transmissivity and, in practice, it is probably not necessary to consider other values of α. The value of $\alpha = 10^{-4}$ is typical for confined aquifers unless the radius of the casing r_c is much smaller than the radius of the well, r_w.

The procedure for determining the transmissivity by this method is one of curve matching. Values of s_w/s_o are prepared

TABLE 16

DRAWDOWN AS A FUNCTION OF RECOVERY TIME FOR $\alpha = 10^{-4}$ AND VARIOUS PUMPING TIMES

s_w/s_o	$\frac{Tt_r}{r_c^2}$			
	Value of $\frac{Tt_o}{r^2}$			
	0	2.5	25	250
1.00	0.0	0.0	0.0	0.0
0.95	0.083	0.130	0.175	0.21
0.90	0.195	0.287	0.375	0.438
0.80	0.470	0.601	0.795	0.941
0.70	0.801	0.990	1.305	1.61
0.60	1.21	1.48	1.93	2.49
0.50	1.72	2.09	2.72	3.72
0.40	2.39	2.83	3.77	5.50
0.30	3.33	3.94	5.40	8.75
0.25	4.00	4.80	6.65	11.9
0.20	4.82	5.95	8.40	18.0
0.15	5.90	7.70	11.0	---
0.10	7.60	10.6	15.3	---

as a function of recovery time from the measured values of water level during recovery. The value of s_o is the drawdown at the beginning of recovery. These results are plotted on semi-log paper with s_w/s_o on the coordinate scale and recovery time on the log scale. This plot is superimposed upon the response function curves shown in Figure 1, and adjusted laterally until a "best fit" is achieved. The best fit may be achieved with a curve that is visually interpolated between the curves shown. Once the best fit is achieved, the dimensionless recovery time that corresponds to a particular recovery time is noted and the transmissivity is computed.

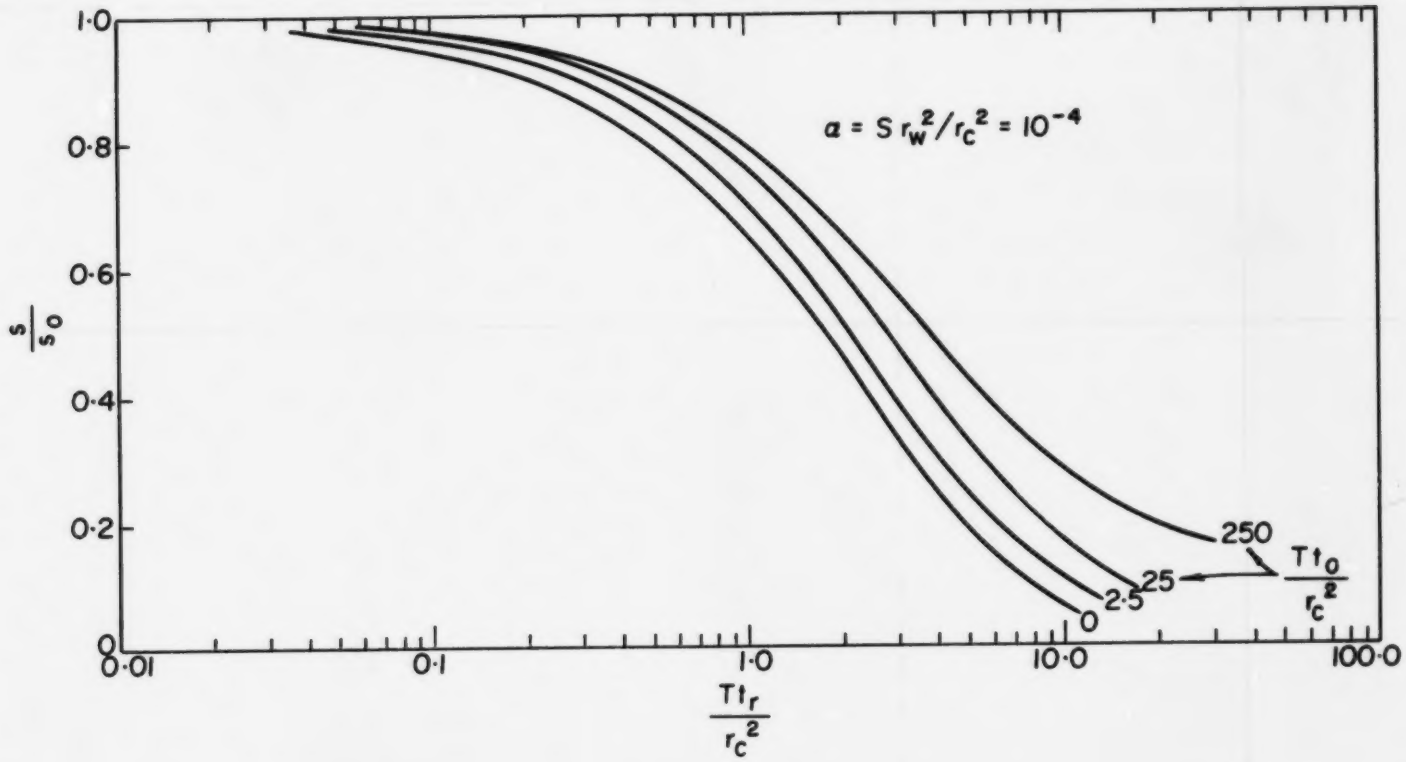

Figure 1. Dimensionless Recovery Functions for $\alpha = 10^{-4}$.

Example #1

The recovery data measured during a single-well aquifer test in a coal hydrology study are tabulated below, along with other pertinent information concerning the test.

r_w = 4.87 cm, r_c = 4.61 cm, t_o = 320 minutes, Q = 10.2 ℓ/min

Total vol. pumped = 3220 ℓ, vol. pumped from Csg = 104 ℓ,

Vol. pumped from aquifer = 3116 ℓ, s_o = 15.542 m

RECOVERY TIME, MIN	0.57	1.05	1.55	2.10	3.42	4.50	5.60	6.48
s_w/s_o	0.950	0.909	0.861	0.818	0.721	0.653	0.592	0.549

7.63	8.45	10.27	12.20	13.48	15.68	20.42	26.02	33.75
0.500	0.470	0.412	0.363	0.338	0.300	0.248	0.209	0.176

38.62	45.10
0.162	0.148

The response functions of Fig.1 are shown superimposed upon these data in Fig. 2. A dimensionless recovery time of 0.40 corresponds to an actual recovery time of 1 min for what is judged to be the "best" fit. As shown on the figure, the calculated transmissivity is 8.5 cm^2/min. The best fit is judged to be on a visually interpolated curve between the curves for which the dimensionless pumping times are 25 and 250. Once the data have been matched to a particular curve and the transmissivity calculated, it is possible to calculate the dimensionless pumping time and check if it corresponds with the curve used for the match. For the example at hand, the dimensionless pumping time is Tt_o/r_c^2 = (8.5) (320)/$(4.61)^2$ = 128 which corresponds to a curve between the two shown.

It is instructive to analyze these data as if the recovery resulted from an instantaneous drawdown of 15.542 m (i.e., as a slug test). Using response function curves prepared from the table presented by Cooper et al. (1967) and superimposing them on the data of this example yields an estimated transmissivity equal to 4.6 cm^2/min; a value which is substantially too low. Furthermore, it is not possible to find a satisfactory match; the data plot indicating a less rapid rate of rise than predicted by any of the response curves. The less rapid rate of rise, of course, is because the gradients causing inflow to the well

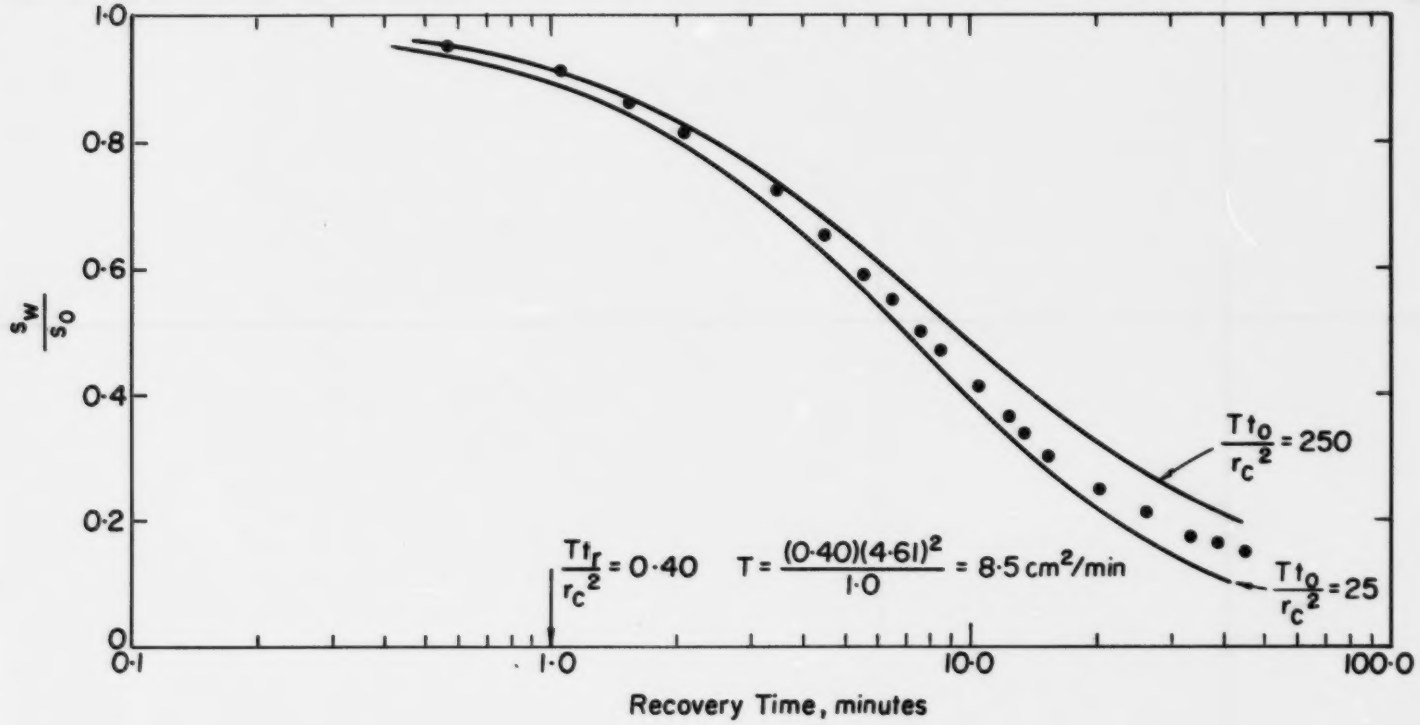

Figure 2. Superposition of Response Functions on Data Plot for Example 1.

are much less in this test than those which would be produced by very rapidly drawing the water level down by 15.542 m. The less rapid rise of the water level, which is actually due to smaller gradients, is approximately accounted for by a reduced transmissivity when the slug test response functions are used.

It is also constructive to analyze the data of this example using the standard recovery analysis in which the drawdown is plotted against the time variable t/t_r on semi-log paper and the transmissivity calculated from the slope of the resulting line. The data and analyses are shown in Fig. 3. Clearly the data do not form a single straight line and two interpretations are possible. The steeper of the two lines yields an estimated T much too low and even the line with a small slope yields a value that is somewhat low. The failure of the standard analysis to provide reliable estimates in this case is attributable to afterflow discharge which is of the same order as the aquifer discharge during the pumping interval. The effects of afterflow become smaller with increasing recovery time and the standard analysis yields increasingly better results as shown by the line with the smaller slope. In many cases, the slope of the data plot is ill-defined at large recovery time, however, and use of this method is not recommended.

Example #2

The recovery data in this example is also from a coal hydrology study, but the transmissivity is approximately an order of magnitude smaller than for Example #1.

r_w = 4.87 cm, r_c = 4.61 cm, t_o = 1508 minutes, Q = 2.24 ℓ/min

Total vol. pumped = 3381 ℓ, vol. pumped from Csg = 172 ℓ,

Vol. pumped from aquifer = 3209 ℓ, s_o = 25.774 m

RECOVERY TIME, MIN	4.00	6.50	9.40	14.00	19.20	32.00	40.40
s_w/s_o	0.960	0.937	0.913	0.878	0.842	0.763	0.724

52.00	67.10	100.40	127.80	191.80	237.00	1368.0
0.665	0.606	0.503	0.438	0.332	0.281	0.038

The data and response curves are superimposed as in Example #1 and the transmissivity is determined to be 0.81 cm^2/min.

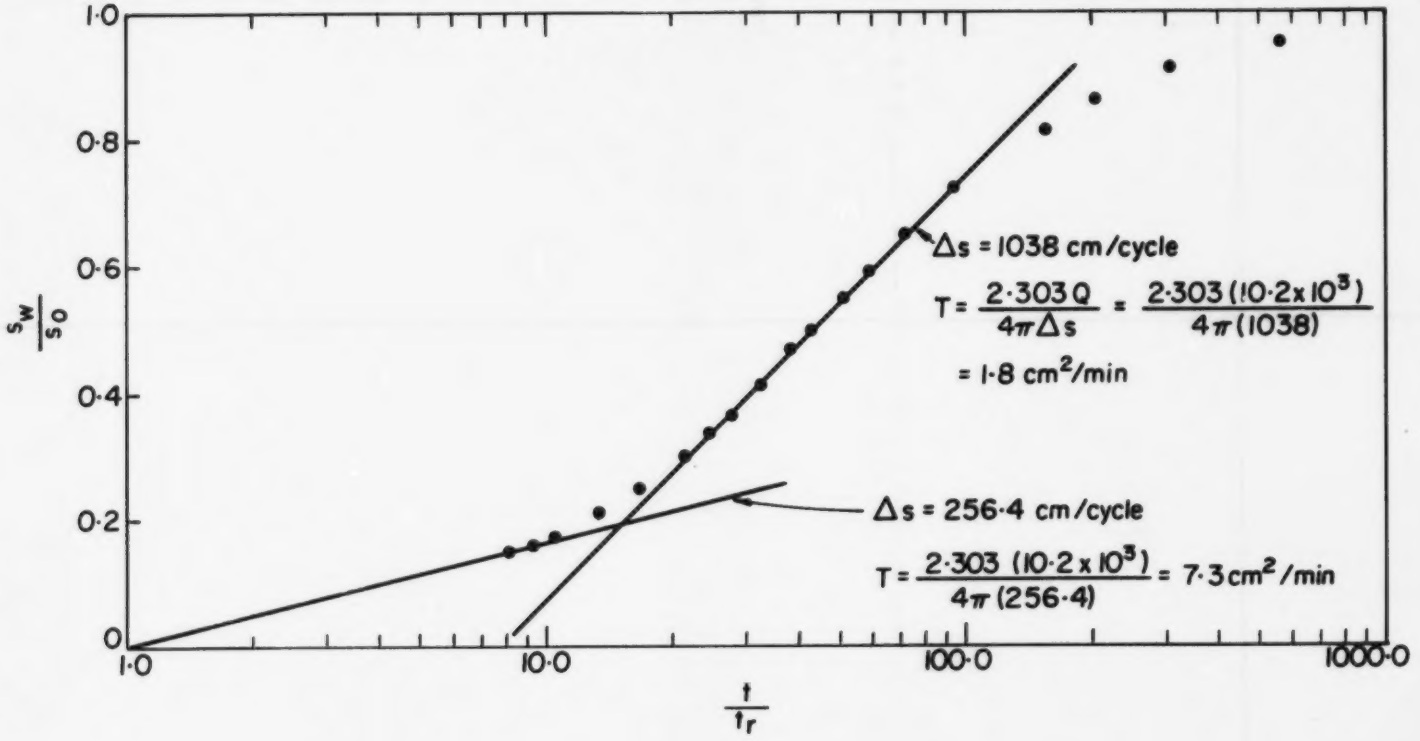

Figure 3. Standard Recovery Analysis of Data in Example 1.

Analysis of this same data as if it were a slug test yields the too low value of T = 0.36 cm^2/min.

ALTERNATE METHOD OF ANALYZING RECOVERY DATA

In those cases where the volume of water pumped from the aquifer is 70 percent or more of the total volume pumped, the standard recovery theory can be modified to account, approximately, for the effects of afterflow. Figure 4 shows a schematic of the aquifer discharge to the well-bore. The declining discharge indicated after pumping ceases is the afterflow.

The basis for the analysis procedure to be developed here is the principle of superposition that permits the drawdown in response to a time varying discharge to be calculated by convolution. An appropriate form for a discretized variation in aquifer discharge is given by McWhorter and Sunada (1977) and by Matthews and Russell (1967).

$$s = \frac{1}{4\pi T} \sum_{i=1}^{n} \Delta Q_i \int_{\beta_i}^{\infty} \frac{\exp(-\chi)}{\chi} d\chi, \quad t \geq t_n \tag{15}$$

where $$\beta_i = \frac{Sr_w^2}{4T(t-t_i)}$$

$$\Delta Q_i = Q_i - Q_{i-1}$$

For the purposes of this analysis, the integral in Eq. 15 will be approximated by $\ln (1/\beta_i) - 0.5772$.

If it is supposed that aquifer discharge truly becomes zero at $t = t_o$, then the drawdown at $t = t_1$ is given by

$$s_1 = \frac{Q_o}{4\pi T_1} \left\{ \ln \frac{4Tt_1}{Sr_w^2} - 0.5772 \right\} - \frac{Q_o}{4\pi T_1} \left\{ \ln \frac{4T(t_1-t_o)}{Sr_w^2} - 0.5772 \right\} \tag{16}$$

or $$s_1 = \frac{Q_o}{4\pi T_1} \ln \frac{t_1}{t_1-t_o}, \tag{17}$$

from which one can compute the transmissivity T, using the measured value of drawdown at time t_1. However, since aquifer discharge did not cease at t_o, the transmissivity so calculated will not be the true value and, therefore, is given the subscript

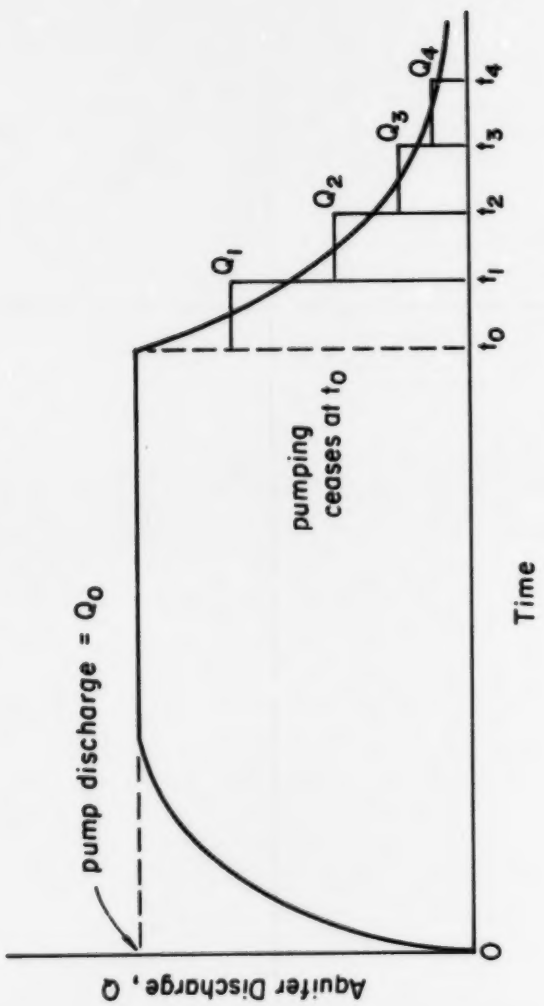

Figure 4. Schematic Drawing of Aquifer Discharge as a Function of Time

corresponding to the time for which it is computed. It is not expected that T_1 is a good approximation to the actual transmissivity because a very significant afterflow has been neglected.

It is now supposed that aquifer discharge ceases at $t = t_1$ and the drawdown at $t = t_2$ is given by

$$s_2 = \frac{Q_o}{4\pi T_2} \ln \frac{t_2}{t_2 - t_o} + \frac{Q_1}{4\pi T_2} \ln \frac{t_2 - t_o}{t_2 - t_1} \qquad (18)$$

obtained by application of Eq. 15 and the logarithmic approximation to the integral. Again, T_2 can be calculated from Eq. 18 by using the measured drawdown at $t = t_2$ and a value of Q_1 calculated from the average rate of rise of the water level in the well during the period from t_o to t_1. Explicitly $Q_1 = \pi r_c^2 (s_o - s_1)/(t_1 - t_o)$ which is generalized to become

$$Q_i = \pi r_c^2 \frac{(s_{i-1} - s_i)}{t_i - t_{i-1}} . \qquad (19)$$

As before, it is not expected that T_2 is a good approximation to the true value but it does represent an improved estimate as compared to T_1. This is true because a somewhat less significant afterflow was neglected.

If the above process is continued up to $t = t_n$ and the result solved for T_n:

$$T_n = \frac{1}{4\pi s_n} \left\{ Q_o \ln \frac{t_n}{t_n - t_o} + \sum_{i=1}^{n-1} Q_i \ln \frac{t_n - t_{i-1}}{t_n - t_i} \right\} . \qquad (20)$$

As explained previously, the T_n calculated from Eq. 20 for each n are only approximate because the afterflow has been truncated in each case. The larger the value of n, the better the approximation and the nearer the calculated T will be to the actual value. This is because the magnitude of the afterflow that is neglected becomes less with increasing n.

Two questions immediately arise: 1) what accuracy can be expected from this analysis and 2) how is the accuracy affected by the size of time steps and the number of steps included in the computation. These questions were investigated by generating recovery data from the exact analytical solution presented

previously in this report, calculating T from Eq. 20 for various sizes of the time increment and for various recovery times, and comparing the calculated T with the actual value used to generate the data. It is concluded that:

a) t_n should be the time at which recovery is at least 90 percent complete.

b) the recovery time considered (i.e. $t_n - t_o$) should be divided into a minimum of 5 equal time intervals.

c) in tests which 80 percent of the total volume pumped is derived from the aquifer, and when guidelines a and b are followed, the approximate calculation will introduce an error of less than 10 percent.

The above statements are demonstrated for a particular case using the exact solution to generate recovery data under the following conditions:

$T = 15.7\ cm^2/min$, $t_o = 100$ min, $Q_o = 14.16\ \ell/min$, $r_w = r_c = 7.925$ cm and $S = 10^{-4}$.

For this situation, the volume pumped is 1416 ℓ, the volume pumped from casing storage is 183 ℓ, and the volume pumped from the aquifer is 1233 ℓ or 87 percent of the total volume pumped. The drawdown data as a function of time using 10-minute time intervals are shown in Table 17. The discharge values also shown in the table were calculated from Eq. 19.

TABLE 17

RECOVERY DATA GENERATED FROM EXACT SOLUTION - 10-MIN INTERVALS

Time, min	s, cm	Q, cm^3/min
100	926.6	14160
110	487.0	8674
120	297.8	3733
130	208.1	1769
140	152.9	1090
150	120.6	637
160	94.7	510

From Eq. 20, the transmissivity is calculated:

$$T = \frac{1}{4\pi(94.7)} \left\{ 14160 \ln \frac{160}{60} + 8674 \ln \frac{60}{50} + 3733 \ln \frac{50}{40} + \ldots \right.$$
$$\left. + 637 \ln \frac{20}{40} \right\} = 14.9 \text{ cm}^2/\text{min}.$$

This value of T is just 5 percent less the exact value of 15.7 cm^2/min.

Table 18 shows the corresponding data using 5 minute time increments.

TABLE 18

RECOVERY DATA GENERATED FROM EXACT SOLUTION - 5-MIN INTERVALS

Time, min	δ, cm	Q, cm^3/min
100	926.6	14160
105	656.1	10672
110	487.0	6675
115	372.5	4517
120	297.8	2948
125	246.2	2036
130	208.1	1501
135	178.7	1161
140	152.9	1020
145	134.9	708
150	120.6	567
155	107.7	509
160	94.7	508

The calculated value of T is:

$$T = \frac{1}{4\pi(94.7)} \left\{ 14160 \ln \frac{160}{60} + 10672 \ln \frac{60}{55} + 6675 \ln \frac{55}{50} + \ldots \right.$$
$$\left. + 509 \ln \frac{10}{5} \right\} = 15.1 \text{ cm}^2/\text{min},$$

which is very close to the value calculated using 10-minute time intervals.

Example #3

The procedures of this section will now be used to determine T for the data given in Example #1. The drawdown data is first plotted to facilitate determining the drawdown at selected values of time. The results for 5 minute intervals are shown in Table 19.

TABLE 19

RECOVERY AND AFTERFLOW DISCHARGE FOR TEST OF EXAMPLE #1

Time, min	s, cm	Q, cm^3/min
320	1554.2	10200
325	972.9	7763
330	660.5	4172
335	481.8	2386
340	390.1	1225
345	335.7	726
350	296.9	518
355	267.3	395
360	247.1	270
365	230.0	228
370	216.0	187

The transmissivity calculated from Eq. 20 is:

$$T = \frac{1}{4\pi(216)} \left[10200 \ln \frac{370}{60} + 7763 \ln \frac{50}{45} + \ldots + 228 \ln \frac{10}{5} \right]$$

$$= 8.4 \text{ cm}^2/\text{min}.$$

This value compares to 8.5 cm^2/min obtained by the curve fitting method determined in Example #1.

Example #4

The data of Example #2 are analyzed by use of Eq. 20 using 100-minute time intervals. The data are shown in Table 20.

TABLE 20

RECOVERY AND AFTERFLOW DISCHARGE FOR TEST OF EXAMPLE #2

Time, min	s, cm	Q, cm^3/min
1508	2577.4	2240.0
1608	1306.7	848.0
1708	829.9	318,0
1808	605.7	149.7
1908	463.9	94.7
2008	366.0	65.4
2108	304.1	41.4
2208	250.6	35.7

The value of transmissivity calculated from Eq. 20 is

$$T = \frac{1}{4\pi(250.6)} \left\{2240 \ln \frac{2208}{700} + 848 \ln \frac{700}{600} + \ldots + 41.4 \ln \frac{200}{100}\right\}$$

$$= 0.9 \text{ cm}^2/\text{min}$$

This value compares with 0.81 cm^2/min determined by curve matching in Example #2.

The above developments and example applications demonstrate how recovery data from single-hole aquifer tests can be analyzed to provide estimates of transmissivity. The developments of this section do not overcome some of the disadvantages inherent in single-hole tests. For example, single-hole test data rarely provide a reliable estimate for storage coefficient and may be significantly affected by well losses. Well losses in properly constructed wells penetrating low transmissivity aquifers can be expected to be less important than in aquifers with high transmissivity, however. Single-hole test data do not provide information on anisotropy or leakage through aquitards. When these are important, multi-well tests must be used.

SECTION 6

ANALYSIS OF SELECTED FLOW PROBLEMS IMPORTANT IN SURFACE MINING HYDROLOGY

Aspects of ground-water hydrology that may be important during the mining phase are several and include: 1) the quality and quantity of inflows to pits, shafts or other excavations, 2) the resultant lowering of the piezometric surface in affected aquifers, 3) inflow to the mine from fault zones, 4) the lowering of water levels in infrequently recharged alluvial aquifers adjacent to the mine, and 5) sustained inflows from frequently recharged alluvial aquifers adjacent to the mine. In this section, the ground-water hydraulics of several such problems are analyzed. Each mine encounters problems that are specific to that particular project, and it is not possible to provide general solutions that will be applicable everywhere. The intent of this section is to provide approximate analytic analyses and solutions that are specifically oriented toward problems known to have been encountered in surface mining projects.

FLOW TO AN ADVANCING PIT

The case analyzed in this sub-section is that in which a surface mine is to be initiated by excavating the box cut through aquifer materials that extend laterally to great distances from the area to be mined. The major questions to be addressed are the inflows to the first cut as the pit advances and the extent to which the piezometric surface is lowered in affected aquifers. The analysis for a particular aquifer intersected by the pit is given here. This aquifer may be the coal seam itself or any other water-bearing stratum between the coal seam and the land surface. If more than one independent aquifer is incised by the pit, the analysis can be applied to each aquifer, individually.

Figure 5 shows an idealized cross-section drawn in a plane normal to the axis of the advancing pit. Only one side is shown in the figure; it being assumed that the conditions are identical on both sides of the pit. The initial piezometric surface is height H_o above the top of the aquifer. When the pit incises the aquifer, a portion becomes unconfined adjacent to the pit and the height of the water table above the bottom of the aquifer is h in this unconfined portion. The distance to which the

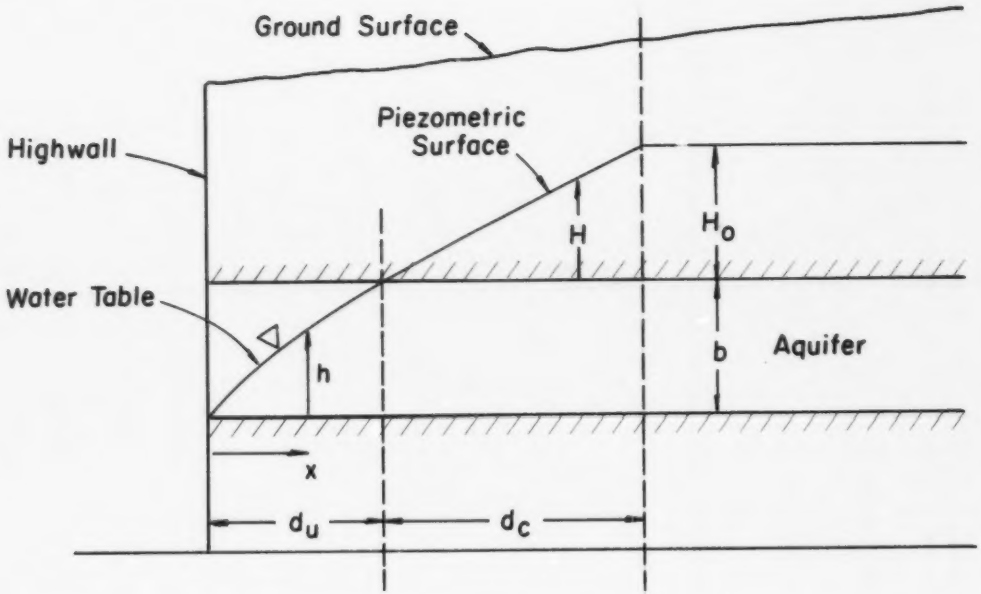

Figure 5. Definition Sketch for Flow to the First Cut.

unconfined portion extends into the aquifer, d_u, becomes larger with increasing time. The fact that the pit is advancing with time is ignored for the moment and flow to the pit in the plane shown in Fig. 5 is analyzed.

The procedure used to obtain a solution is that of a succession of steady states. In this method, the discharge is assumed to be independent of the space variable at a particular instant in time, thus permitting one to calculate the distribution of piezometric head as if the flow were steady. As the discharge changes with time, the distribution of head also changes, but in accordance with that predicted from the appropriate steady state equation. This method is approximate, of course, but has the important property that mass continuity is accounted for exactly. The method has been used by Bear (1972), Polubarinova-Kochina (1952) and Arawin and Numerov (1953, 1965), among others.

At a particular instant in time, the volume of water that has been produced into the pit in a unit length of pit is given by the exact expression

$$V_d = S_{ya} \int_0^{d_u} (b-h)\,d\chi + S \int_{d_u}^{d_c+d_u} (H_0-H)\,d\chi \qquad (1)$$

where V_d = volume drained to pit per unit length of pit

S_{ya} = apparent specific yield

S = storage coefficient

b = aquifer thickness

h = height of the water table above floor of aquifer

H_0 = piezometric height above top of aquifer

d_u = extent of unconfined zone

d_c = distance between the boundary of the confined and unconfined zone and the point at which $H = H_0$

In the unconfined zone, the flow is given by

$$q = Kh \frac{dh}{d\chi} \qquad 0 \leq \chi \leq d_u \qquad (2)$$

where q = discharge to the pit per unit length of pit

K = hydraulic conductivity (permeability)

and other symbols are as defined previously. The succession of steady states permits Eq. 2 to be integrated

$$q \int_0^{\chi} dx = K \int_0^{h} h \, dh \tag{3}$$

or

$$h = \left(\frac{2q}{K}\right)^{\frac{1}{2}} \chi^{\frac{1}{2}} \qquad 0 \leq \chi \leq d_u \tag{4}$$

For $\chi = d_u$, Eq. 3 gives

$$q = \frac{Kb^2}{2d_u} \tag{5}$$

In a similar way, the distribution of head in the confined zone is computed to give

$$H = \frac{q}{T}(x - d_u), \quad d_u \leq \chi \leq d_u + d_c \tag{6}$$

and

$$q = \frac{T H_0}{d_c} \tag{7}$$

Equations 4 and 6 are substituted into Eq. 1 and the integrations performed to yield

$$V_d = S_{ya} \frac{bd_u}{3} + S \frac{d_c H_0}{2} \tag{8}$$

The variables d_u and d_c in Eq. 8 are replaced in terms of q by substituting with Eqs. 5 and 7 resulting in

$$V_d = \left\{\frac{S_{ya} Tb^2}{6} + \frac{STH_0^2}{2}\right\} \frac{1}{q} \tag{9}$$

It is now noted that $dV_d/dt = q$ so that Eq. 9 can be written as the ordinary differential equation

$$q = -\left\{\frac{S_{ya} Tb^2}{6} + \frac{STH_0^2}{q^2}\right\} \frac{1}{q^2} \frac{dq}{dt} \tag{10}$$

Integration subject to $q = \infty$ at $t = 0$ yields

$$q = \left\{ \frac{S_{ya} Tb^2}{12} + \frac{STH_0^2}{4} \right\}^{\frac{1}{2}} t^{-\frac{1}{2}} . \qquad (11)$$

Should the first cut pit be established in its entirety at the instant $t = 0$, then Eq. 11 could be multiplied by the length of the pit to yield the inflow discharge from one side. The entire pit is not established instantaneously, of course, and the discharge so calculated would be much too large. To account for the fact that the length of pit increases with time, it is noted that Eq. 11 gives the discharge per unit length of pit from a slice of aquifer that became exposed in the pit at time zero. Thus, in Eq. 11, t is the time interval over which the aquifer has been exposed in the pit. This time interval is more conveniently expressed by $t-\tau$ wherein t is time measured from the initiation of pit excavation and τ is the time at which a particular slice of aquifer becomes exposed in the pit. Hence,

$$q = A\,(t-\tau)^{-\frac{1}{2}} \qquad (12)$$

where $$A = \left\{ \frac{S_{ya} Tb^2}{12} + \frac{STH_0^2}{4} \right\}^{\frac{1}{2}} .$$

The differential discharge to the pit from a differential slice of aquifer is, therefore:

$$dQ = q\,dy = A\,(t-\tau)^{-\frac{1}{2}}\,dy \qquad (13)$$

where y is a coordinate measure along the pit axis from the point of beginning, and dQ is the differential discharge.

It is supposed that the length of pit increases linearly with time at an average rate of advancement equal to R. In this case

$$y = R\tau \qquad (14)$$

from which $$dy = R\,d\tau \qquad (15)$$

and Eq. 13 becomes $$dQ = A\,(t-\tau)^{-\frac{1}{2}}\,R\,d\tau . \qquad (16)$$

The discharge to the pit is obtained by integrating over τ from 0 to t to yield

$$Q = 2R \left\{ \frac{S_{ya} Tb^2}{12} + \frac{STH_0^2}{4} \right\}^{\frac{1}{2}} t^{\frac{1}{2}} , \quad t \leq \frac{L}{R} \qquad (17)$$

which holds for all times during the advancement of the pit (i.e. for $t \leq L/R$, where L is the maximum length of the pit).

Once the pit has reached its maximum length and elongation ceases, the discharge is calculated by integrating Eq. 16 over τ from 0 to L/R and results in

$$Q = 2R \left\{ \frac{S_{ya} Tb^2}{12} + \frac{STH_0^2}{4} \right\}^{\frac{1}{2}} \left\{ t^{\frac{1}{2}} - (t-L/R)^{\frac{1}{2}} \right\}, \quad t \geq \frac{L}{R} \qquad (18)$$

The discharge to the pit decreases sharply once the maximum length has been attained as will be shown in a subsequent example. The discharge in Eqs. 17 and 18 must be multiplied by 2 to obtain the discharge from both sides of the cut.

All of the equations required to calculate the distance to which the piezometric surface remains undisturbed are now available. Specifically the distances d_u and d_c are given by

$$d_u = \frac{Tb}{2} \left\{ \frac{S_{ya} Tb^2}{12} + \frac{STH_0^2}{4} \right\}^{-\frac{1}{2}} (t-y/R)^{\frac{1}{2}}, \quad \frac{y}{R} \leq t \qquad (19)$$

and

$$d_c = TH_0 \left\{ \frac{S_{ya} Tb^2}{12} + \frac{STH_0^2}{4} \right\}^{-\frac{1}{2}} (t-y/r)^{\frac{1}{2}}, \quad \frac{y}{R} \leq t \qquad (20)$$

The total distance to the edge of the disturbed zone is the sum of d_u and d_c.

Probably the most limiting aspect in the foregoing analysis is the implicit assumption that flow occurs only in planes normal to the long axis of the advancing cut. This is not strictly true, of course, but permits the estimation of discharge in a relatively simple manner that does account for all the major, first-order factors influencing the discharge. The discharge to the pit from aquifers exposed in the highwall as calculated from the above equations will be conservatively large.

Example #1

The box cut of a surface coal mine will incise an unconfined aquifer perched above the target coal seam. The saturated thickness of the aquifer is 18 m and the transmissivity and apparent specific yield are 0.92 m^2/d and 0.05, respectively. The mine is expected to produce coal at the rate of 3.62×10^5 m^3/yr from a pit that is 30 m in width penetrating a coal seam that averages 2.09 m thick. Under these circumstances the average rate of advance of the box cut is

$$R = \frac{3.62 \times 10^5 \ m^3/yr}{(365 \ d/yr)(30 \ m)(2.09 \ m)} = 15.8 \ m/d \ .$$

The maximum length of the pit is 915 m. The inflow to the box cut and the distance to which the water table will be effected as functions of time are desired.

Because the aquifer is unconfined, the parameter H_0 is zero in all of the equations developed above. Other parameters are $T = 0.92\ m^2/d$, $S_{ya} = 0.05$, $b = 18$ m, $L = 915$ m, and $R = 15.8$ m/d. The full pit length is achieved in $915/15.8 = 58$ d. The discharge computed from Eq. 17 for $t \leq 58$ d and from Eq. 18 for $t \geq 58$. The results are shown in Figure 6, and include inflow from both sides of the pit.

FLOW TO AN ADVANCING PIT INITIATED ON A CROP LINE

A schematic cross-section of the physical conditions assumed to exist for this case is shown in Figure 7. The first cut is assumed to be made near the crop line and extended to full length along the strike of the coal bed. The second cut is made in the down-dip direction from the first cut and spoil is cast in the up-dip direction. In this case it is necessary to account for both the elongation of the cuts and the fact that the highwall is advancing down-dip with time. An important effect of the sloping aquifer is that the drawdown s_0 (i.e. the difference between the initial piezometric surface elevation and the head at the face of the highwall) increases incrementally as the highwall advances down-dip.

If the water-bearing stratum intersected by the cut is an unconfined aquifer then in an aquifer slice normal to the cut, the volume drained per unit length of pit is

$$V_d = S_{ya} \int_0^{d_u} s \, d\chi \ . \qquad (21)$$

where s is the drawdown as shown in Fig. 7. Under consideration now is the box cut only, or actually, the first for which the bottom of the pit will be below the initial water table elevation. The drawdown produced at the highwall by this cut is s_{01}. If the effect of the sloping aquifer floor on the saturated thickness h is neglected, the distribution of s with χ can be calculated as in the previous subsection, substituted into Eq. 21, and the integration carried out to yield

$$V_d = S_{ya} \frac{s_{01} d_u}{3} \ . \qquad (22)$$

Also

$$d_u = \frac{K(s_{01})^2}{2q} \qquad (23)$$

The drawdown s_{01} is a constant for the first cut so the

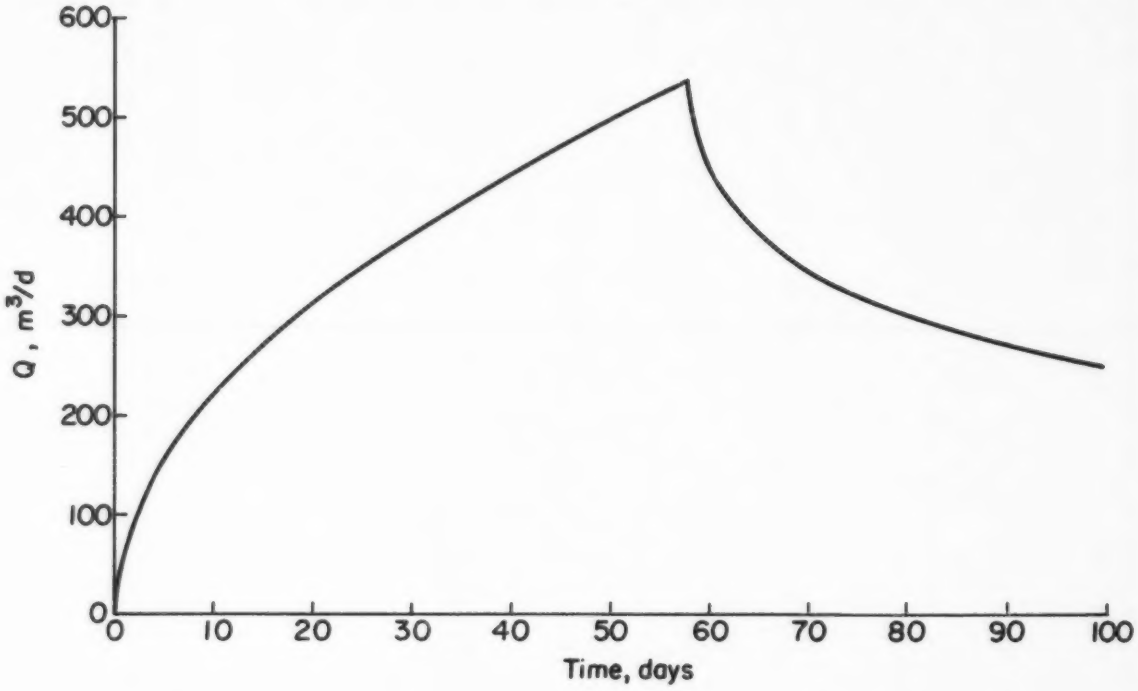

Figure 6. Calculated Inflows to Box Cut for Example 1.

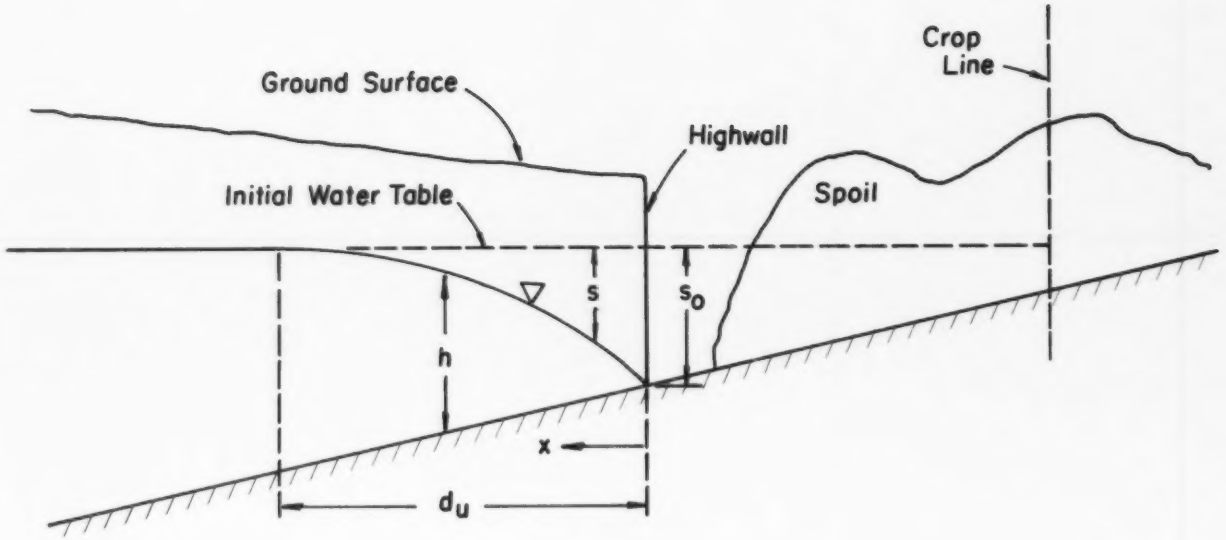

Figure 7. Schematic Physical Model for Flow to a Pit Initiated on a Crop Line.

results of the previous sub-section apply and the discharge to the first cut is

$$Q_1 = 2R\left\{\frac{S_y a^T \delta^2_{01}}{12}\right\}^{\frac{1}{2}} t^{\frac{1}{2}}, \quad t \leq \frac{L_1}{R_1} \tag{24}$$

and

$$Q_1 = 2R\left\{\frac{S_y a^T \delta^2_{01}}{12}\right\}^{\frac{1}{2}} \left\{t^{\frac{1}{2}} - (t-L_1/R_1)^{\frac{1}{2}}\right\}, \quad t \geq L_1/R_1. \tag{25}$$

where L_1 equals length of the first cut and R_1 is the rate at which it is elongated.

The second cut will be made down-dip from the first one and will produce an additional increment of drawdown δ_{02}. The additional discharge to the pit due to this additional increment of drawdown is calculated from a suitable modification of Eqs. 24 and 25. For simplicity in notation, let the time required to complete the first cut be $t_1 = L_1/R_1$, the time required to complete the second cut be $t_2 = L_2/R_2$, and so on for each cut. With this notation the incremental discharge due to the second cut is

$$Q_2 = 2R\left\{\frac{S_y a^T \delta^2_{02}}{12}\right\}^{\frac{1}{2}} (t-t_1)^{\frac{1}{2}} \quad t_1 \leq t \leq t_1 + t_2 \tag{26}$$

and

$$Q_2 = 2R\left\{\frac{S_y a^T \delta^2_{02}}{12}\right\}^{\frac{1}{2}} \left\{(t-t_1)^{\frac{1}{2}} - (t-t_1-t_2^{\frac{1}{2}}\right\} \quad t \geq t_1 + t_2 \tag{27}$$

In a similar way the incremental discharge for each successive cut is calculated. The total discharge at any time is the sum of the incremental discharges for each cut. The incremental drawdown caused by advancing the pit down-dip is computed from

$$\delta_0 = W \tan \theta \tag{28}$$

where W is the width of the cut and θ is the dip angle.

The calculations outlined above are simplified substantially for the special case in which $t_1 = t_2 \ldots t_n$, and $\delta_{01} = \delta_{02} = \ldots \delta_{0n}$. In this case the total discharge to the pit becomes

$$Q = 2R\left\{\frac{S_y a^T \delta^2_0}{12}\right\}^{\frac{1}{2}} t^{\frac{1}{2}}. \tag{29}$$

Example #2

Mining will be initiated near the crop line of a coal seam that dips 6 degrees. The mine will be located in a valley that narrows in the down-dip direction and the length of successive cuts, oriented parallel to the strike, will become less as the valley narrows. A schematic plan and section view of the area to be mined is shown in Fig. 8. The width of each cut will be 30 m and the rate of increase of length will be 10 m/d. The length of the first cut will be 600 m, and the length of each successive cut will be 30 m less than the previous one. Other required parameters are $S_{ya} = 0.05$, $K = 0.1$ m/d, $T = 2.6$ m^2/d, $\delta_{01} = \delta_{02} = \ldots = 30 \tan 6^0 = 3.153$ m.

Table 21 shows the computation for times up to the completion of the 4th cut. For this case $t_1 = 60$ d, $t_2 = 57$, $t_3 = 54$ $t_4 = 51$ and so on. The discharges indicated under the columns headed Q_3 were computed from

$$Q_3 = 6.563\ (t-117)^{\frac{1}{2}} \qquad 117 \leq t \leq 171$$

$$Q_3 = 6.563\ (T-117)^{\frac{1}{2}} - (t-171)^{\frac{1}{2}} \qquad t \geq 171$$

and Q_4 was computed from

$$Q_4 = 6.563\ (t-171)^{\frac{1}{2}} \qquad 171 \leq t \leq 222$$

FLOW TO A PIT ADVANCING PARALLEL TO AN ALLUVIAL VALLEY

Ground water may be withdrawn from an alluvial aquifer into the mine through permeable strata that connect the mine and the alluvium. An idealzed cross-section is shown in Fig. 9. It is assumed that the transmissivity of the alluvial aquifer is large relative to that of the strata providing hydraulic communication between the alluvium and the mine, so that a constant head is maintained as $x = a$. Unlike the situations analyzed previously, a steady state flow from the alluvium to the mine can be established following a transient period during which the effects of inflow to the mine propagate outward from the highwall. The analysis is carried out by neglecting the transient period.

At steady state the discharge per unit of pit length through the confined portion is equal to that through the unconfined portion so that

$$\frac{Kb^2}{2d_u} = \frac{TH_0}{a-d_u} \qquad (30)$$

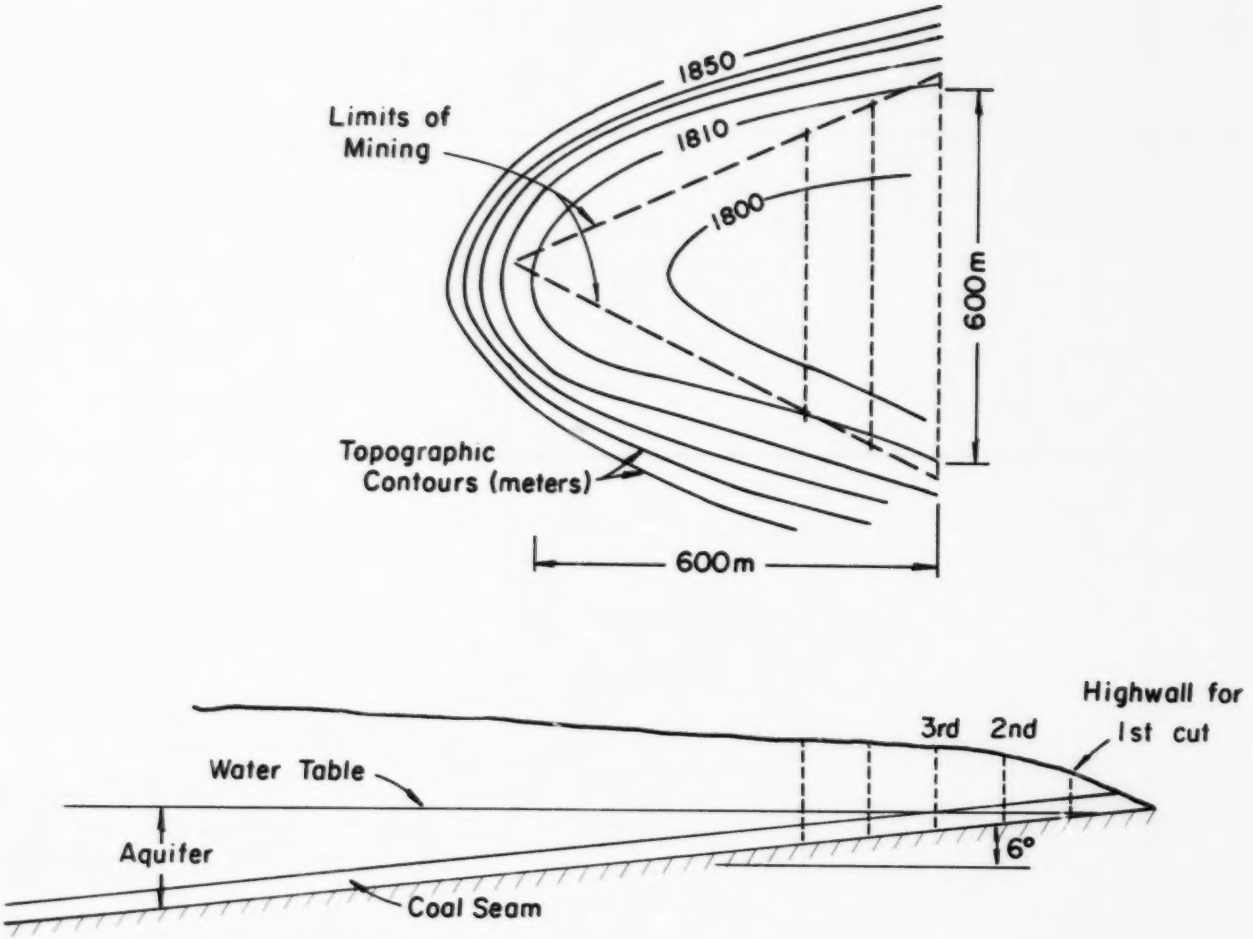

Figure 8. Schematic of the Physical Conditions for Example 2.

TABLE 21

COMPUTATION OF DISCHARGE TO THE MINE OF EXAMPLE #2

CUT NO.	TIME DAYS	Q_1 EQ. 24	Q_1 EQ. 25	Q_2 EQ. 26	Q_2 EQ. 27	Q_3	Q_3	Q_4	Q_{total} m^3/d
1	0								0
	16	26							26
	36	39							39
	60	51	0	0					51
2	70		34	21					55
	90		26	36					62
	117		21	50	0	0			71
3	132		20		30	25			75
	152		18		24	39			81
	171		17		21	48	0	0	86
4	190		16		19		27	29	91
	210		15		17		22	41	95
	222		14		16		20	47	97

from which

$$d_u = \frac{ab}{2H_0+b} \tag{31}$$

so that the expression for the steady discharge per unit of pit length is

$$q_s = \frac{Tb}{2}\left(\frac{2H_0+b}{ab}\right) \tag{32}$$

where q_s is the steady discharge per unit of pit length. As done previously

$$dQ = q_s\, dy = \frac{Tb}{2}\left(\frac{2H_0+b}{ab}\right) R\, d\tau \tag{33}$$

or

$$Q = \frac{RTb}{2}\left(\frac{2H_0+b}{ab}\right) t, \qquad t \leq \frac{R}{L} \tag{34}$$

where R is the average rate of advance of the pit. Discharge to

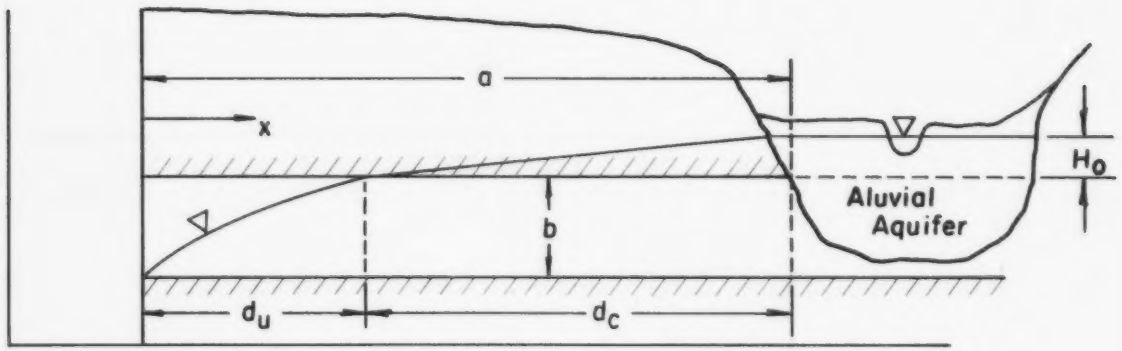

Figure 9. Schematic Drawing of Flow to a Mine from an Alluvial Aquifer.

the pit becomes constant after the maximum length is attained and is given by

$$Q = \frac{RTb}{2}\left(\frac{2H_0+b}{ab}\right)\frac{L}{R} \quad . \tag{35}$$

Discharge to the pit will be somewhat greater during the transient phase than predicted by the above developments for the steady phase. The approximate time period required for the steady state to develop can be estimated by calculating the time at which the transient discharge per unit of pit length becomes equal to the steady value. From Eq. 11 with q given by Eq. 32

$$t_s = \left\{\frac{2ab}{Tb(2H_0+b)}\right\}^2 \left\{\frac{S_{ya}Tb^2}{12} + \frac{STH_0^2}{4}\right\} \tag{36}$$

where t_s is the time required to reach steady state after a particular point has been exposed in the pit.

Example #3

A stratum 11 m thick with a transmissivity of 2.5 m^2/d subcrops in the alluvium associated with a perennial stream. The stratum will be cut through by the pit of a surface mine. The pit will be advanced to a maximum length of 600 m parallel to the stream at a distance of 50 m from the edge of the alluvial aquifer. The elevation of the water table in the alluvium is 3 m greater than the elevation of the top of the subcropping stratum. Estimate the maximum steady discharge form the alluvium to the pit.

The maximum steady discharge will occur when the pit has reached its maximum length and is given by

$$Q = \frac{Tb}{2}\left(\frac{2H_0+b}{ab}\right)L$$

$$= \frac{(2.5)(11)}{2}\left(\frac{2(3)+11}{(50)(11)}\right)600$$

$$= 255\ m^3/d \ .$$

DRAINAGE OF ALLUVIUM OR FAULT ZONE RECEIVING NO RECHARGE

The flow problem under consideration in this subsection is similar to that analyzed in the previous one. If the alluvium

receives no recharge, however, it will drain and does not constitute a constant head boundary. Figure 10 shows the physical model for the problem of interest in this subsection. The infrequently or non-recharged alluvial aquifer shown in Fig. 10 could also be a fault zone or even a lake or reservoir.

If the short period of transient flow, in which water is derived from storage in the connecting stratum, is neglected then from the previous subsection

$$q = \frac{T}{a}\left(\frac{b}{2} + H\right), \qquad H \geq 0 \tag{37}$$

where all symbols are as previously defined. In Eq. 37, the head H is a function of time. The flow through the connecting stratum as given by Eq. 37 must also be given by

$$q = - W S_{ya} \frac{dH}{dt} \tag{38}$$

where W is the width and S_{ya} is the apparent specific yield of the alluvial aquifer or fault zone. If the water is supplied by a lake or reservoir, then $S_{ya} = 1$. Equation 38 expresses relationship between the decline of the water level and the rate of drainage. Equating the discharges in Eqs. 37 and 38 and integrating the resulting ordinary differential equation yields

$$\left(\frac{b}{2} + H\right) = \left(b/2 + H_0\right) \exp\left(\frac{-Tt}{aWS_{ya}}\right) \tag{39}$$

where H_0 is the initial value of H. The result in Eq. 39 is substituted into Eq. 37 to yield

$$q = \frac{T}{a}\left(\frac{b}{2} + H_0\right) \exp\left(\frac{-Tt}{aWS_{ya}}\right) \tag{40}$$

which gives the discharge per unit length of pit.

The effect of the advancement of the pit is accounted for by replacing t with $t-\tau$ and noting that $dQ = Rq\, d\tau$, where R is the mean rate of elongation of the pit. The results are

$$Q = R\left(\frac{b}{2} + H_0\right) W S_{ya} \left\{1 - \exp\left(\frac{-Tt}{aWS_{ya}}\right)\right\}, \quad t \leq \frac{L}{R} \tag{41}$$

and

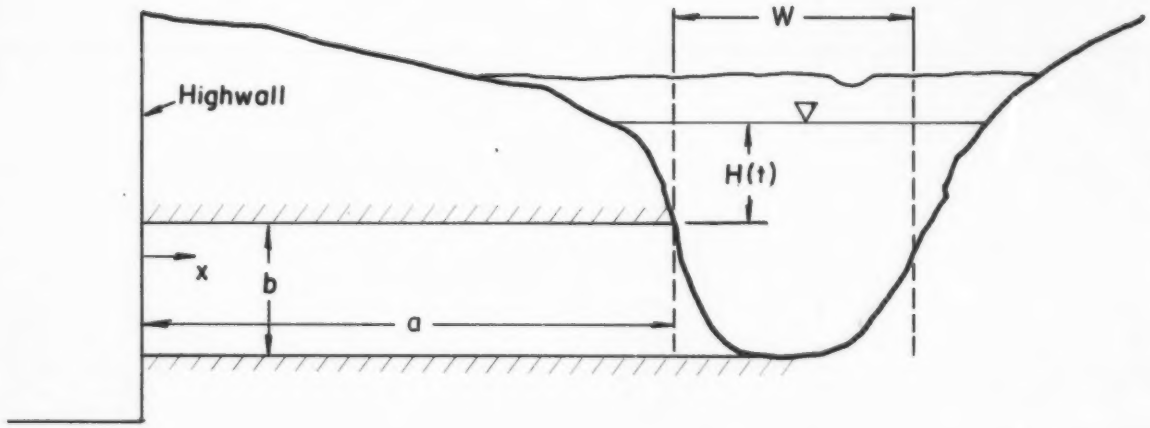

Figure 10. Schematic of Physical Model for Drainage of an Aquifer or Fault Zone.

$$Q = R\left(\frac{b}{2} + H_0\right) W S_{ya} \left\{\exp\left[-\frac{T(t-L/R)}{aWS_{ya}}\right] - \exp\left(\frac{-Tt}{aWS_{ya}}\right)\right\}, \quad t \geq \frac{L}{R} . \tag{42}$$

Example #4

The equations developed above can be used to compute the rate of filling of a mined area just as well as the drainage of an aquifer or fault zone. Consider the physical situation shown in Fig. 11, in which flow is occurring from the alluvial aquifer into the backfilled mine area. This flow will cease when the water table in the spoil has risen to the elevation of that in the alluvial aquifer.

For the following conditions, estimate the time required for the water table in the spoils to rise to within 1 m of the elevation of the water table in the alluvial aquifer: H_0 = 20 m, T = 2.5 m^2/d, S_{ya} (spoils) = 0.1, a = 50 m, and the slope of the impervious layer is 8 degrees from the horizontal.

In this problem, no consideration of an advancing pit is required and Eq. 39 is the appropriate starting point. Because the aquifer is unconfined, b = 0 and T is an average value being equal to $KH_0/2$. Also, the value of W to be used in the computations must be an average value. The appropriate average value of W is given by

$$\overline{W} = \frac{1}{2}\frac{H_0}{\tan 8^0} = 71 \text{ m.}$$

When the water table in the spoils is within 1 m of that in the alluvium, H = 1 m. Thus, from Eq. 39

$$\frac{Tt}{aWS_{ya}} = \ln \frac{H_0}{H}$$

or $$t = \frac{(50)(71)(0.1)\ \ln\left(\frac{20}{1}\right)}{2.5} = 425 \text{ days.}$$

DISCHARGE FROM SPOIL IN RESPONSE TO VERTICAL RECHARGE

Under consideration in this subsection is the accumulation and discharge of ground water from a spoil bank in response to vertical infiltration. The physical model is shown schematically in Fig. 12. Solutions to the linearized Boussinesq equation

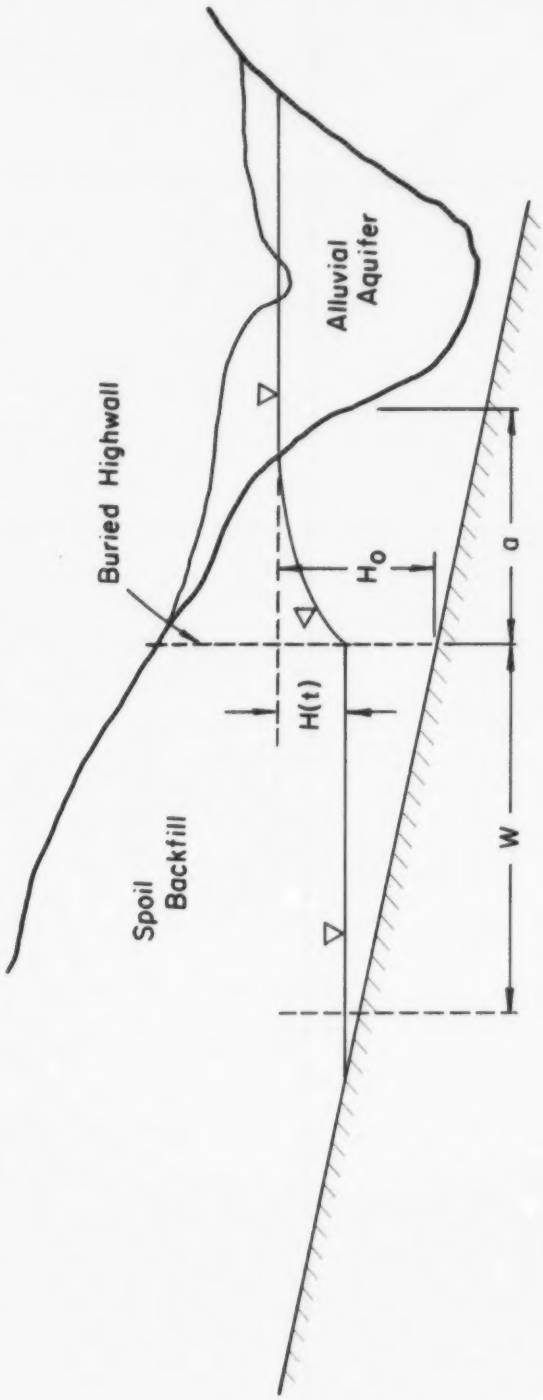

Figure 11. Schematic Physical Model for Example 4.

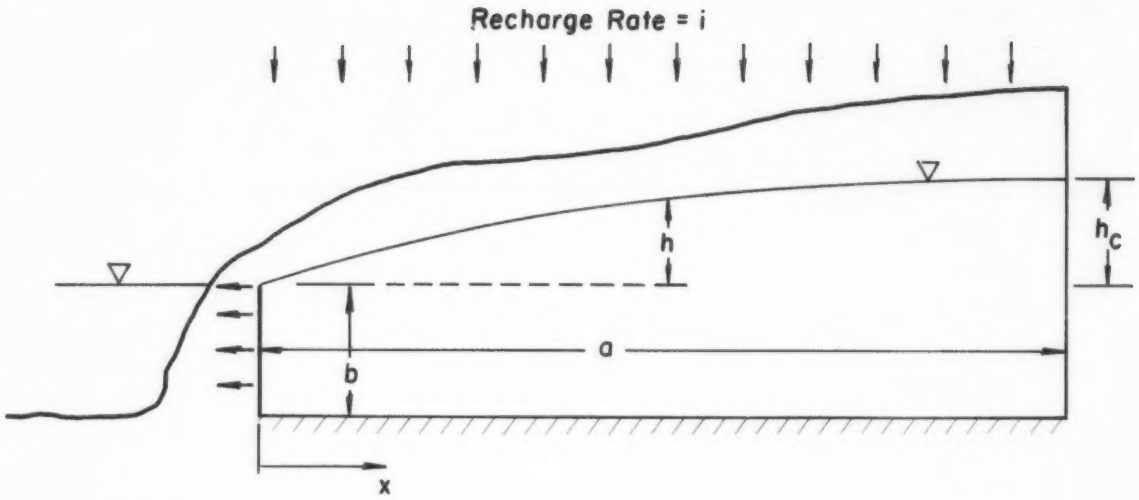

Figure 12. Physical Model for the Buildup and Discharge of Ground Water in a Spoil Bank.

provide approximate results of engineering utility for this case. The linearized Boussinesq equation is

$$\frac{T}{S_{ya}} \frac{\partial^2 h}{\partial \chi^2} = \frac{\partial h}{\partial t} \tag{43}$$

where T is the product of permeability K and an averaged saturated thickness that will be calculated subsequently.

A fundamental solution to Eq. 43 for boundary conditions of $h(0,t) = 0$ and $\partial h/\partial \chi = 0$ at $\chi = a$ is

$$h = \frac{4I}{\pi S_{ya}} \sum_{n=1,3,5..}^{\infty} (1/n) \sin \frac{n\pi\chi}{2a} \exp\left(\frac{-n^2 \pi^2 Tt}{4S_{ya} a^2}\right) \tag{44}$$

where I is an increment of recharge (volume per unit area) that reached the water table at $t = 0$. Thus, Eq. 44 represents the fundamental response function to a finite increment of recharge that reaches the water table instantaneously at $t = 0$.

Maasland (1959) showed how the water table response to intermittent recharge can be calculated from Eq. 44. McWhorter (1977) used this approach to calculate the water-table response to an arbitrary sequence of recharge events that occur in a time interval t_o and are then repeated indefinitely. Of particular interest in mine spoil hydrology is the long term pattern of discharge and water table fluctuations that can be anticipated in the spoil bank.

An arbitrary sequence of instantaneous recharge events, I_1, $I_2, \ldots I_m$, are considered. This arbitrary pattern of recharge occurs in one year and is assumed to occur in an indentical fashion in all subsequent years. It can be shown (McWhorter, 1977) that the water table elevation h_c at the point $\chi = a$ is given by

$$h_c = \sum_{i=1}^{m} \frac{I_i}{S_{ya}} M_i \tag{45}$$

where

$$M_i = \frac{4}{\pi} \sum_{n=1,3,5...}^{\infty} (-1)^{(n-1)/2} \left\{ \frac{\exp\left[-(\mu_n t_0) \frac{\tau_i}{t_0}\right]}{1-\exp(-\mu_n t_0)} \right\} \tag{46}$$

$$\mu_n = \frac{n^2\pi^2 T}{4a^2 S_{ya}} \qquad (46)$$

t_0 = period of repeating recharge pattern = 1 year

and τ_i is the time interval between the time of interest and the time at which event i last occurred. Thus, the time variable τ_i can take on all values between 0 and t_0.

The discharge per unit length of spoil bank can be obtained using the above theory and is (McWhorter, 1977)

$$q = \frac{2T}{aS_{ya}} \sum_{i=1}^{m} I_i \sum_{n=1,3,5..}^{\infty} \left\{ \frac{\exp\left[-(\mu_n t_0 \tau_i / t_0)\right]}{1-\exp(-\mu_n t_0)} \right\} . \qquad (47)$$

For values of $(T\tau_i)/4a^2 S_{ya}$ greater than about 0.06, the first terms in the infinite series in Eqs. 46 and 47 dominate and the approximate discharge formula

$$q = \frac{\pi T h_c}{2a} \qquad (48)$$

is derived by combining Eqs. 46 and 47 using the first terms only. The mean value of transmissivity, calculated from a time and space weighted averaging procedure (McWhorter, 1977) is given by

$$T = \frac{Kb}{2} + \frac{1}{2}\left\{(Kb)^2 + \frac{4Pa^2K}{3}\right\}^{\frac{1}{2}} \qquad (49)$$

where P = average recharge rate $= \left(\sum_{i=1}^{m} I_i\right) / t_0$.

Figure 13 is a plot of the response function M_i for various values of the parameter $Tt_0/4a^2 S_{ya}$. In most applications it will be found sufficiently accurate to compute the discharge from Eq. 48 after h_c has been computed from Eq. 45 using Fig. 13. The procedure is demonstrated in the following example.

Example #5

Analysis of precipitation and meteorological records, coupled with a water balance calculation on the root zone of a spoil bank, yield the following estimates for long term mean recharge

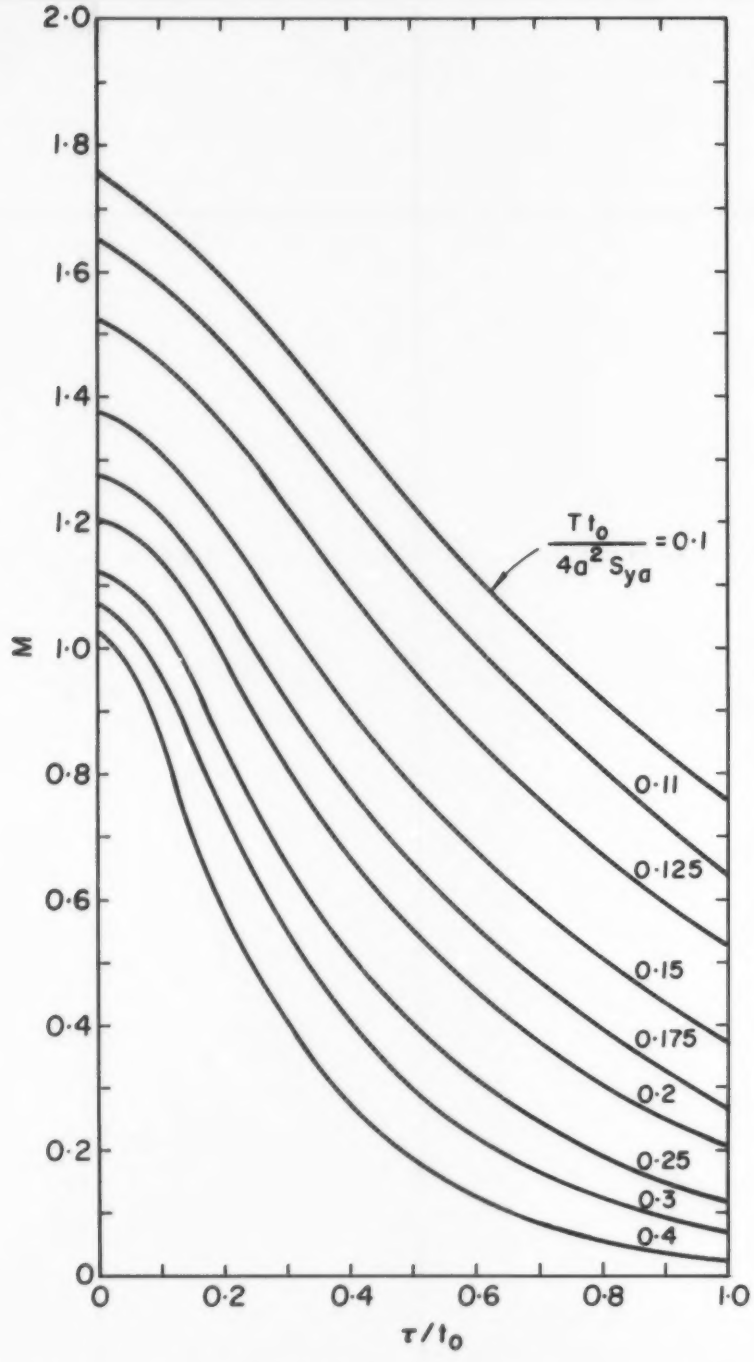

Figure 13. Dimensionless Water Table Height Resulting from Periodic Recharge of Magnitude I_i.

in the reclaimed mine: I_1 = 10 cm in April and I_2 = 4 cm in May. The relatively large recharge in April is due to snow melt and recharge in May is due to rain during the spring period when the moisture content in the root zone is still high from the snow melt. In this particular example, precipitation events during other periods of the year are insufficient to bring the depleted soil moisture content up to a level where deep percolation occurs. All infiltration from such events is consumed by evapotranspiration. All of the estimated recharge in April is assimulated into a single value, assumed to occur instantaneously on April 15. Likewise, the entire 4 cm of May recharge is assumed to occur instantaneously on May 25. As explained previously, these are regarded as long mean values that are assumed to repeat year after year.

The spoil bank for this problem is in the form of a rectangle 360 m long (L = 360 m) and 100 m wide (a = 100 m). The permeability of the spoil is 1.3×10^{-1} m/d, S_{ya} = 0.05, and b = 2 m. The first step in the computation is to calculate the mean transmissivity from Eq. 49 with P = (10 + 4)/365 = 0.0411 cm/d = 4.11×10^{-4} m/d.

$$T = \frac{(1.3\times10^{-1})}{2} + \frac{1}{2}(2.6\times10^{-1})^2 + \left\{\frac{4(4.11\times10^{-4})(100)^2(1.3\times10^{-1})}{3}\right\}^{\frac{1}{2}} = 0.57 \text{ m}^2/\text{d}.$$

Therefore, the curve parameter is

$$\frac{Tt_0}{4a^2S_{ya}} = \frac{(0.57)(365)}{4(100^2(0.05)} = 0.1$$

The computation of discharge is demonstrated in Table 22.

The calculation was begun on April 15, immediately following recharge event number one. Thus, zero time had elapsed since event one occurred, but 325 days had elapsed since the May 25 recharge event occurred in the previous year. On April 25, ten days had elapsed since the April 15 event and 335 days since the May 25 event. Once τ_1 and τ_2 were determined for each date, the ratios τ_1/t_0 and τ_2/t_0 were computed and the corresponding values of M_1 and M_2 read from Fig 13. The values of h_{c1} were obtained by multiplying M_1 by I_1/S_{ya} and h_{c2} =

TABLE 22

DISCHARGE FROM SPOIL BANK FOR EXAMPLE #5

Date	τ_1 Days	M_1	h_{c1} m	τ_2 Days	M_2 m	h_{c2} m	h_c m	Q m^3/d
Apr. 15	0	1.755	3.51	325	0.842	0.67	4.18	13.5
25	10	1.735	3.47	335	0.820	0.66	4.13	13.3
May 5	20	1.715	3.43	345	0.800	0.64	4.07	13.1
15	30	1.695	3.39	355	0.780	0.62	4.01	12.9
25	40	1.675	3.35	0	1.755	1.40	4.75	15.3
June 4	50	1.650	3.30	10	1.735	1.39	4.69	15.1
14	60	1.630	3.26	20	1.715	1.37	4.63	14.9
24	70	1.602	3.20	30	1.695	1.36	4.56	14.7
July 4	80	1.575	3.15	40	1.675	1.34	4.49	14.5
Aug. 3	110	1.485	2.97	70	1.602	1.28	4.25	13.7
Sept. 2	140	1.375	2.75	100	1.512	1.21	3.96	12.7
Oct. 2	170	1.272	2.54	130	1.415	1.13	3.67	11.8
Nov. 1	200	1.170	2.34	160	1.310	1.05	3.39	10.9
Dec. 1	230	1.085	2.17	190	1.207	0.97	3.14	10.1
31	260	1.000	2.00	220	1.110	0.89	2.89	9.3
Jan. 30	290	0.925	1.85	250	1.028	0.82	2.67	8.6
Mar. 1	320	0.855	1.71	280	0.950	0.76	2.47	8.0
31	350	0.790	1.58	310	0.878	0.70	2.28	7.3

$(I_2/S_{ya})M_2$. Corresponding values of h_{c1} and h_{c2} were added to obtain h_c from which the discharge values were computed using Eq. 48 multiplied by the length of the spoil bank (i.e. 360 m). Use of Eq. 48 to calculate the discharge results in predicted values of q that are less than the values calculated from the more rigorously correct Eq. 47.

One might well question the use of instantaneous recharge events when it is known that the recharge accumulates gradually over a finite period of time. However, the above computation illustrates the tremendous damping effect of the ground-water reservoir. Little substantive change in the discharge hydrograph would result if the same quantities of recharge had been distributed over a period of a few days, but the computation procedure would have been complicated significantly.

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